Challenges and Opportunities in Controlling Electrical Loads

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work undertaken with

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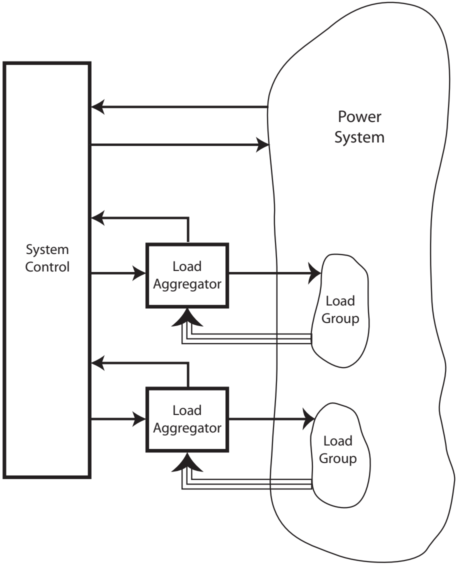
Motivation

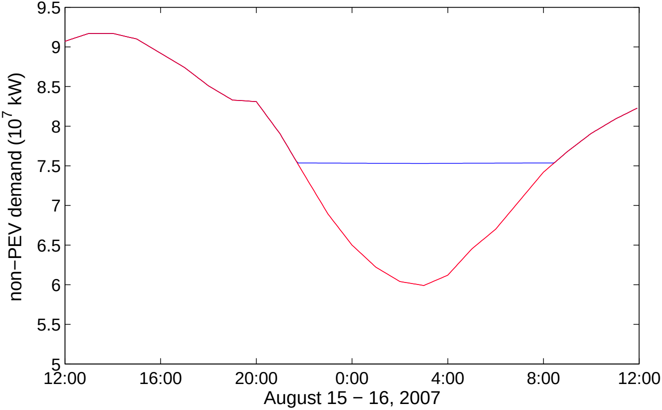
- Current paradigm:
  - Load is an exogenous input.
  - Generation tracks fluctuations.
- This will not work when renewable generation is a significant proportion of total load.
  - Ramp rate limits on generators.
  - Need excessive amount of reserve, which is expensive.
- Ubiquitous communications facilitates control of highly distributed loads.

Hierarchical control structure

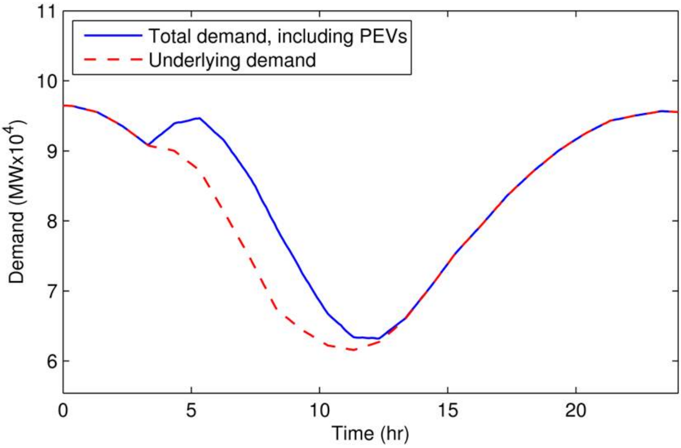


Plug-in electric vehicles

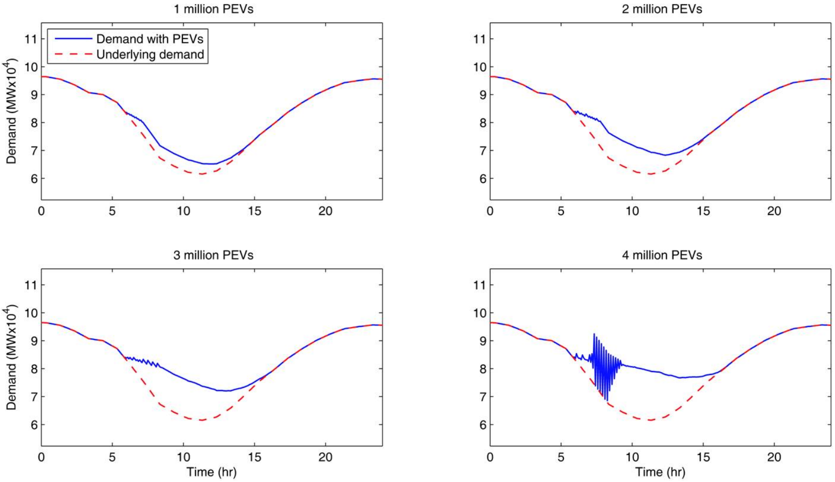
- Charging control strategies will be vitally important for ensuring large-scale adoption of plug-in EVs does not cause generation scheduling problems.



Time-based charging strategy



Price-based charging strategy



Dynamics and notation

- Individual SOC dynamics

$$x_{t+1}^n = x_t^n + \frac{\alpha^n}{\beta^n} u_t^n, \qquad i = 0, ..., T-1$$

N	Size of PEV population
$[0, T]$	Charging interval
β^n	Battery size of PEV n
$u_t^n \geq 0$	Charging rate for PEV n
α^n	Charging efficiency of PEV n
x_t^n	State of charge (SOC) of PEV n at time t
x_0^n	Initial SOC value of PEV n

Large populations

- Assume $N \to \infty$.
- Total electricity generating capacity, $C/N = c$.
- Total non-PEV demand at time t, $D_t/N = d_t$.

- Electricity price function

$$p(\cdot) \equiv p\Big(\frac{D_t + \sum_{n=1}^{N} u_t^n}{C}\Big) = p\Big(\frac{d_t + \mathrm{avg}(\mathbf{u}_t)}{c}\Big).$$

where $\mathrm{avg}(\mathbf{u}_t) \triangleq \frac{1}{N}\sum_{n=1}^{N} u_t^n$.

- Define

$$r_t \equiv \frac{d_t + \mathrm{avg}(\mathbf{u}_t)}{c}.$$

Individual costs

- Agent cost function

$$J^n(\mathbf{u}) \triangleq \sum_{t=0}^{T-1} p(r_t) u_t^n$$

- Individual charging control problem

$$\min_{u^n} J^n(u^n; \mathbf{u}^{-n}),$$

subject to $u_t^n \geq 0$, and $x_T^n = 1$.

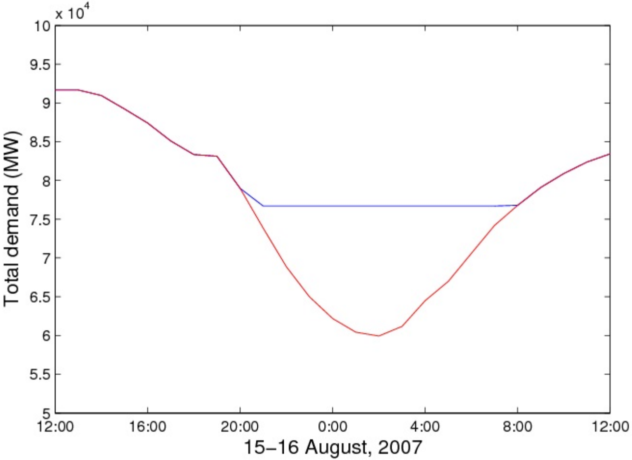
- $\mathbf{u}^*$ is a *Nash equilibrium (NE)* iff

$$J^n(u^{n,*}; \mathbf{u}^{-n,*}) \leq J^n(u^n; \mathbf{u}^{-n,*}),$$

for all u^n and all n.

Nash equilibrium

Main result: The desired valley-filling strategy is given by the unique Nash equilibrium (as N approaches infinity).

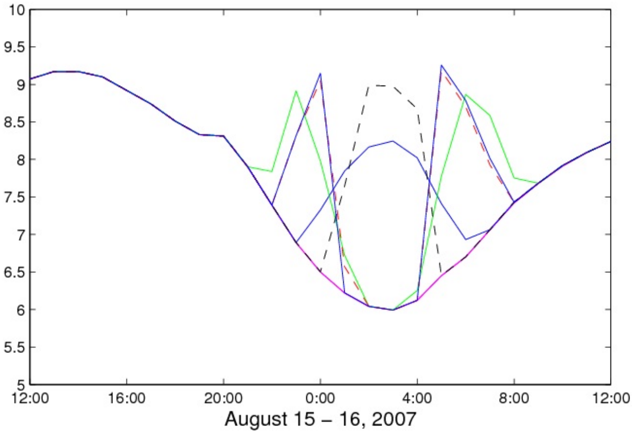


Decentralized update mechanism

The following charging negotiation procedure takes place sometime prior to the actual charging interval.

1. The utility broadcasts base demand d to PEVs.
2. Each PEV proposes its optimal strategy with respect to a common aggregate PEV demand broadcast by the utility.
3. The utility collects all the individual strategies proposed in (2), and updates the aggregate PEV demand accordingly. This updated aggregate demand is rebroadcast to all PEVs.
4. Repeat (2) and (3) until the optimal strategies proposed by the PEVs no longer change.

Non-convergence



Modified (tracking) cost function

To avoid oscillations, we introduce a tracking function:

$$J^n(\mathbf{u}) \triangleq \sum_{t=0}^{T-1} \left\{ p(r_t)u_t^n + \delta\left(u_t^n - \mathrm{avg}(\mathbf{u}_t)\right)^2 \right\}$$

with tracking parameter $\delta > 0$.

Theorem

The collection of charging controls $\mathbf{u} \equiv \{u^n; n < \infty\}$ *is a Nash equilibrium if:*

1. *Every* u^n *is a local control minimizing the cost function,*

$$J^n(u^n;\overline{u}) = \sum_{t=0}^{T-1} \left\{ p(\frac{d_t + \overline{u}_t}{c}) u_t^n + \delta(u_t^n - \overline{u}_t)^2 \right\}$$

   with respect to $\overline{u}$*, and*

2. $\overline{u}_t = avg(\mathbf{u}_t)$.

Local optimal tracking strategy

Define $u_t^n(\overline{u}, A)$ satisfying:

$$u_t^n(\overline{u}, A) = \frac{1}{2\delta} \max \left\{ 0,\ A - p\left(\frac{d_t + \overline{u}_t}{c} \right) + 2\delta \overline{u}_t \right\}$$

Theorem

$u^n(\overline{u}, A)$ is the unique optimal control with respect to $\overline{u}$.

Proof.

Apply the methods of Lagrange multipliers. □

Existence of Nash equilibrium

Theorem

Assume the price function $p(r)$ is continuous on r. Then there exists a Nash equilibrium for the infinite-population decentralized charging control problem.

Proof.

- We can show $u^{n,*}(\overline{u})$ is continuous on $\overline{u}$; then $\mathrm{avg}(\mathbf{u}_t^*(\overline{u}))$ is continuous on $\overline{u}$.
- Hence by Brouwer's fixed point theorem, there exists $\overline{u}$, such that $\mathrm{avg}(\mathbf{u}_t^*(\overline{u})) = \overline{u}$.

□

Uniqueness and convergence of Nash equilibrium

Theorem

Assume $p(r) \in C^1$ and increasing on r, and δ satisfies

$$\frac{1}{2c}\sup\frac{dp}{dr} \leq \delta \leq \frac{a}{c}\inf\frac{dp}{dr}, \quad \textit{with}\ \frac{1}{2} < a < 1. \tag{1}$$

Then the system converges to a unique Nash equilibrium.

Proof.

Under inequality (1),

$$\left|\mathrm{avg}(\mathbf{u}^*(\overline{u})) - \mathrm{avg}(\mathbf{u}^*(\overline{v}))\right|_1 \leq (2 - \frac{1}{a})\left|\overline{u} - \overline{v}\right|_1 < \left|\overline{u} - \overline{v}\right|_1.$$

By the contraction mapping theorem, the system converges to a unique fixed point $\overline{u}$ such that $\mathrm{avg}(\mathbf{u}^*(\overline{u})) = \overline{u}$. □

Theorem

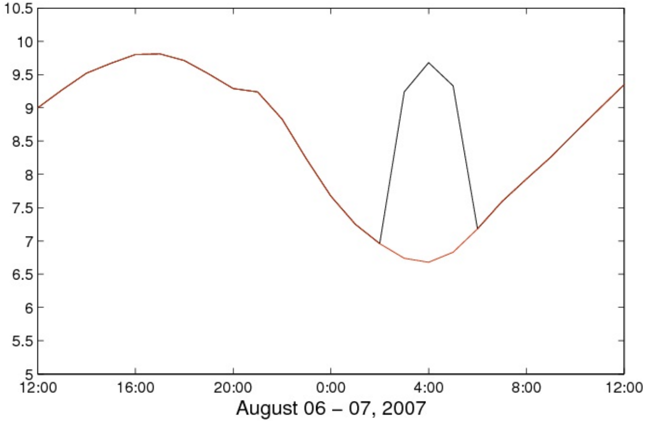
Suppose $\mathbf{u}^*$ is a Nash equilibrium, and p is strictly increasing. Then $\mathbf{u}^*$ satisfies the properties,

$$\begin{aligned} \operatorname{avg}(\mathbf{u}_t^*) &\geq \operatorname{avg}(\mathbf{u}_s^*), && \text{when } d_t \leq d_s, \\ \operatorname{avg}(\mathbf{u}_t^*) + d_t &\leq \operatorname{avg}(\mathbf{u}_s^*) + d_s, && \text{when } d_t \leq d_s, \\ \operatorname{avg}(\mathbf{u}_r^*) + d_r &= B, && \text{for all } r \in [\widehat{t}_0, \widehat{t}_s], \end{aligned}$$

with $[\widehat{t}_0, \widehat{t}_s]$ a sub-interval of the charging period where $u^{n,*} > 0$, for all n.

- For a homogeneous population of PEVs, the above properties correspond to exact valley filling.
- For a heterogeneous population of PEVs, the properties correspond to a strategy that nearly fills the valley.

Homogeneous PEV populations



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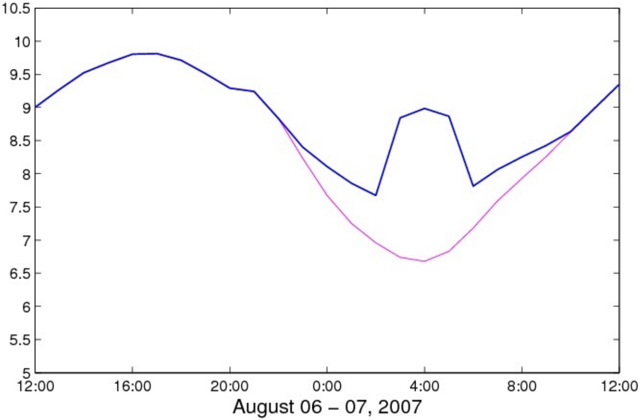

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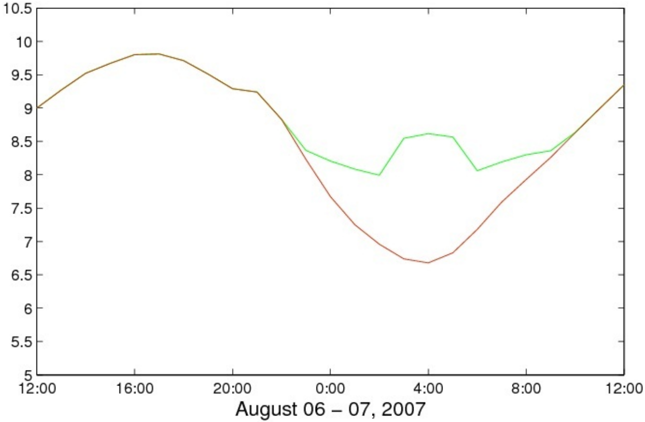

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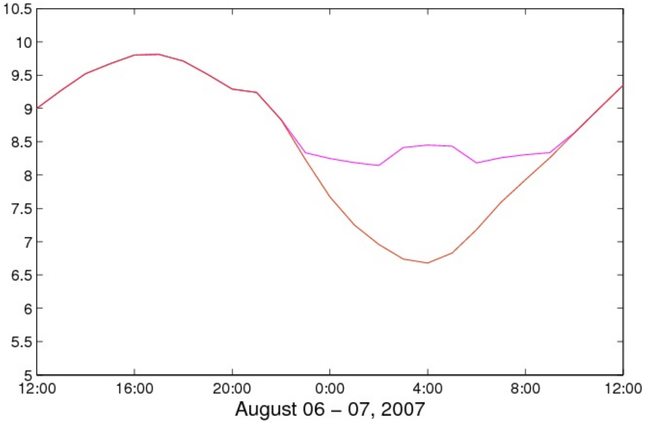

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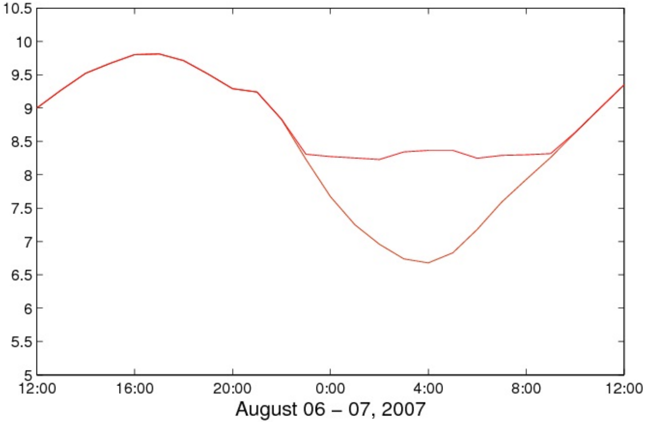

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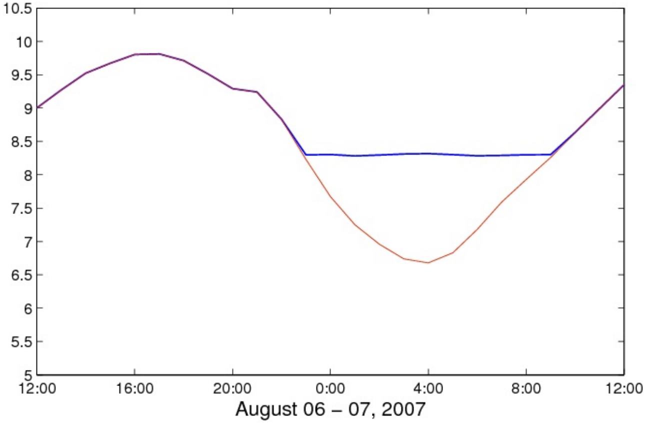

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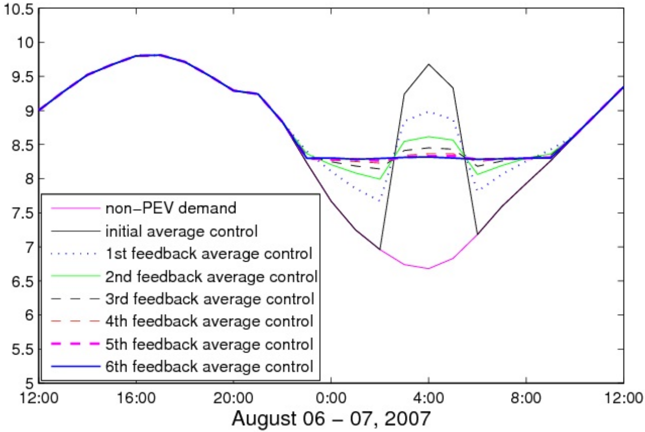
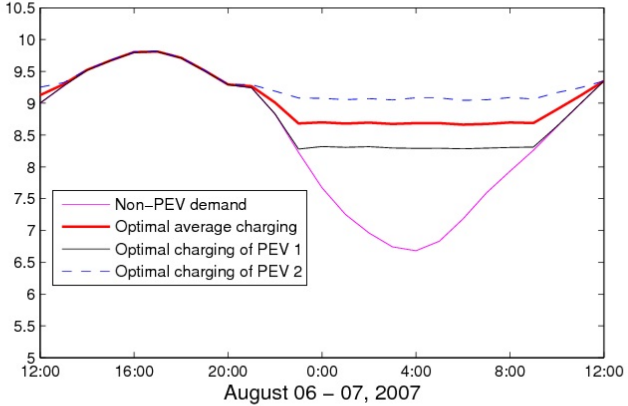
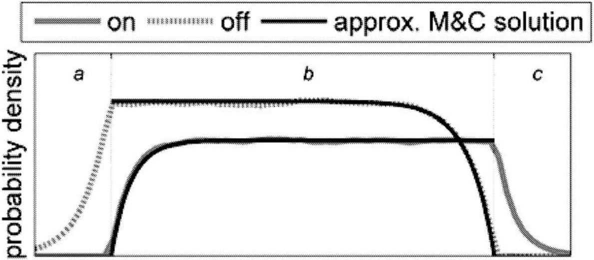


2 classes of PEV populations



Control of air-conditioning load (Callaway)

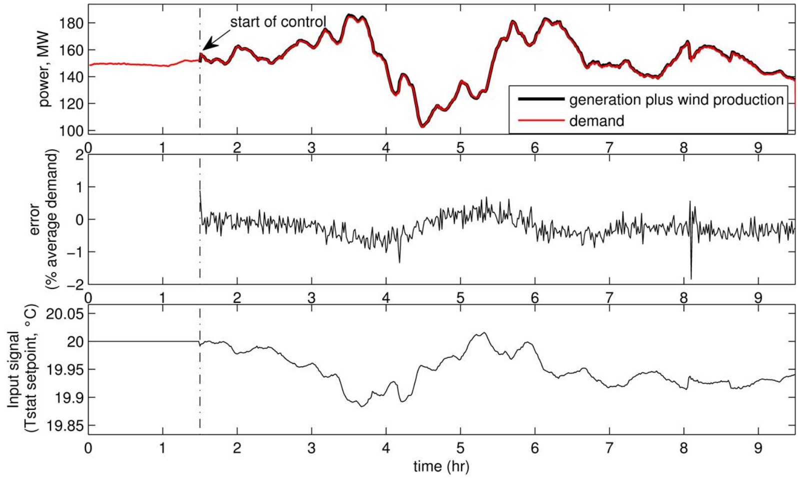
- Steady-state temperature distribution for 10,000 cooling loads.



- Regions:
  - ‘a’ contains only loads in the off state.
  - ‘b’ contains loads in both on and off state.
  - ‘c’ contains only loads in the on state.
- Control strategy:
  - Increase load by lowering setpoint.
  - Decrease load by raising setpoint.

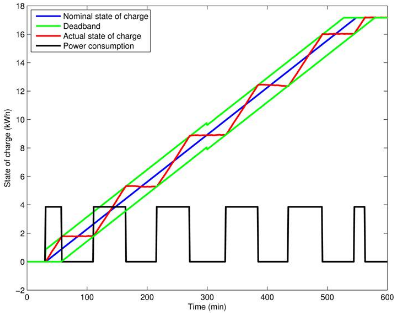
Tracking wind variations

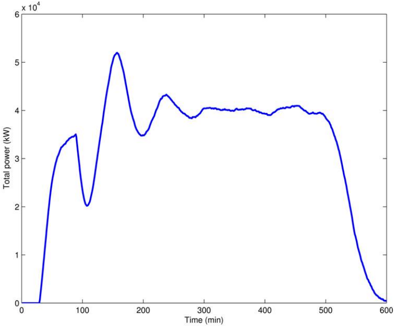
- Controlling air-conditioning loads to follow wind variations.



Hysteresis-based control of PEV load

- Establish a hysteresis band around the nominal charging trajectory.





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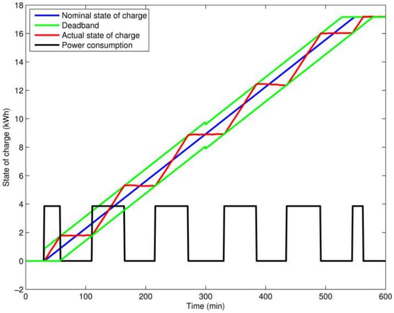
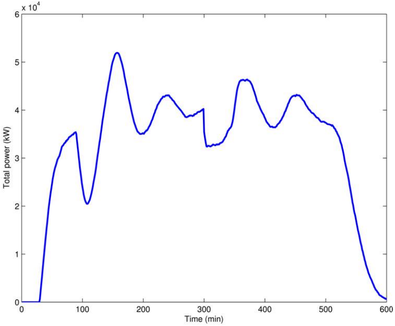
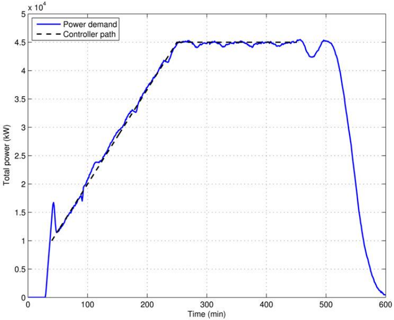
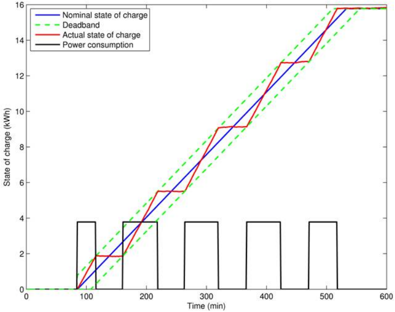




Tracking control

- Total PEV charging load can be forced to track a desired schedule.





Load control: PEV charging

Tracking control

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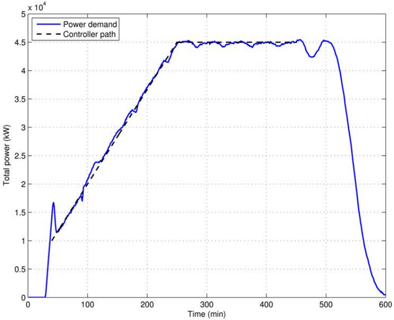

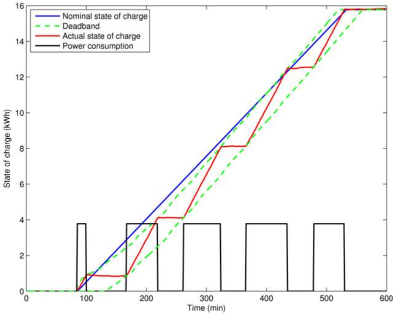

Conclusions

- Significant actuation can be achieved through coordinated control of large numbers of highly distributed loads.
- Issues:
  - Control structure, latency, data security, ...
  - Incentives for consumers to participate in fast-acting demand response schemes.