Optimal Demand Response with Renewables

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Focus: renewable integration

DR model

Preliminary results

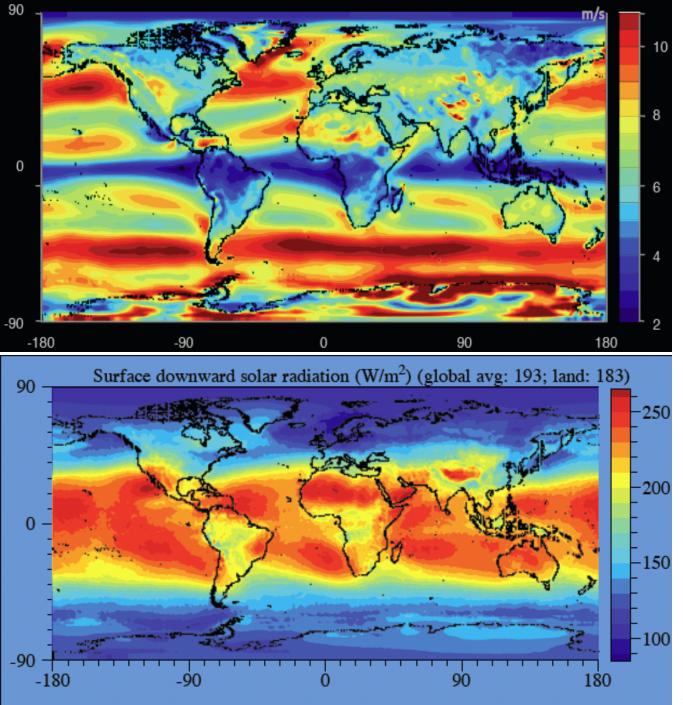


Wind power over land (outside Antartica): 70 – 170 TW

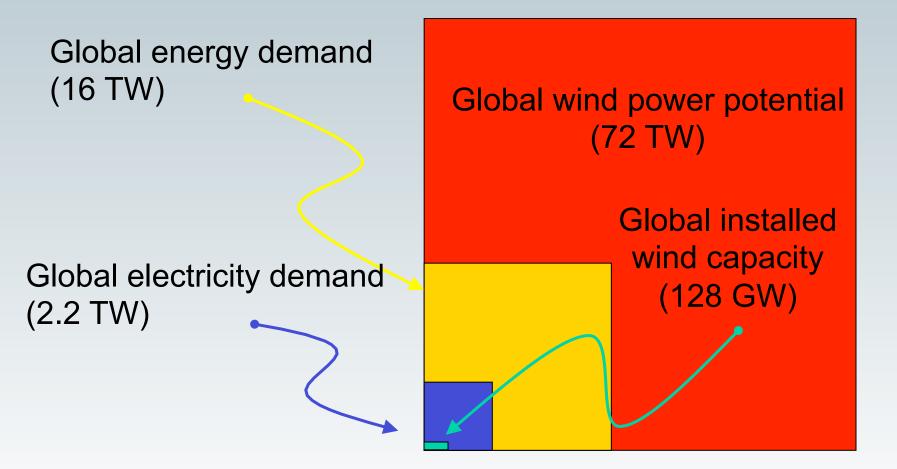
World power demand: 16 TW

Solar power over land: 340 TW

Source: M. Jacobson, 2011

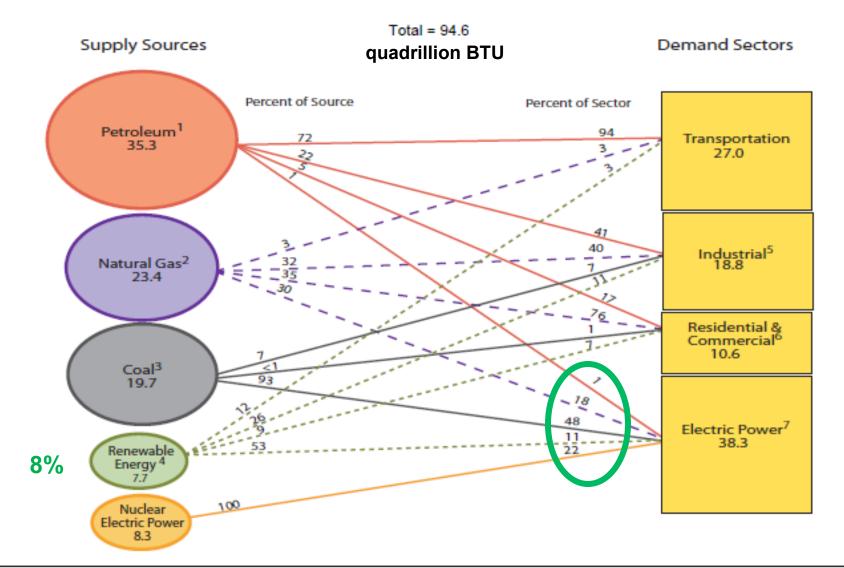


Why renewable integration?



Source: Cristina Archer, 2010

US Primary Energy Flow 2009



¹ Does not include biofuels that have been biended with petroleum—biofuels are included in "Renewable Energy."

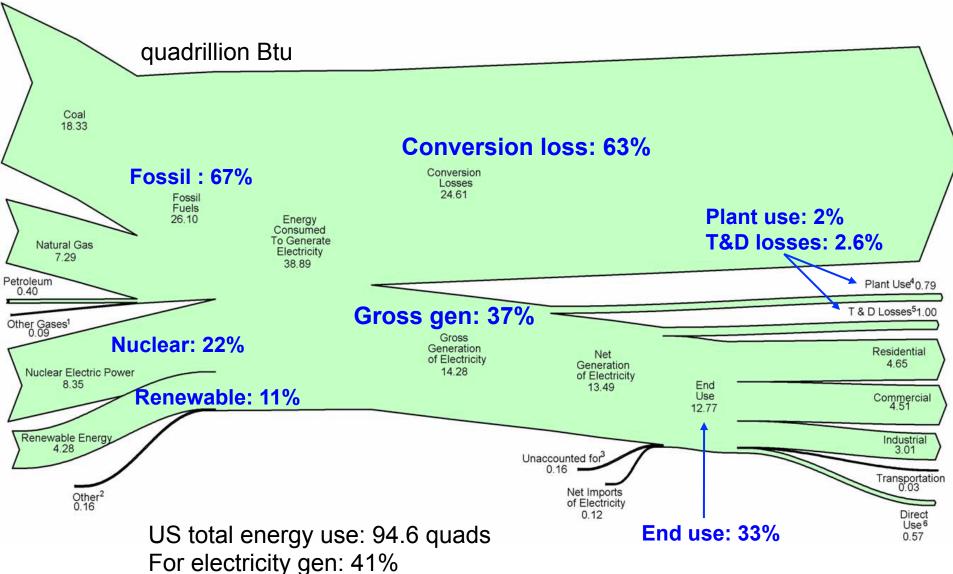
- ² Excludes supplemental gaseous fuels.
- ³ Includes less than 0.1 quadrillon Btu of coal coke net exports.
- 4 Conventional hydroelectric power, geothermal, solar/PV, wind, and biomass.
- ⁶ Includes Industrial combined-heat-and-power (CHP) and Industrial electricity-only plants.

Includes commercial combined-heat-and-power (CHP) and commercial electricity-only plants.

⁹ Electricity-only and combined-heat-and-power (CHP) plants whose primary business is to sell electricity, or electricity and heat, to the public.

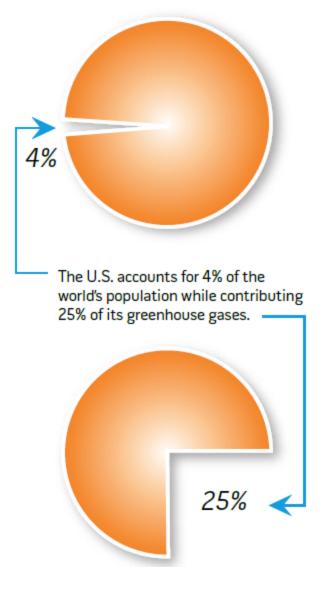
Note: Sum of components may not equal total due to independent rounding. Sources: U.S. Energy Information Administration, Annual Energy Review 2009, Tables 1.3, 2.1b-2.1f, 10.3, and 10.4.

US electricity flow 2009



Source: EIA Annual Energy Review 2009

Generate more than electricity...



US CO₂ emission Elect generation: 40% Transportation: 20%

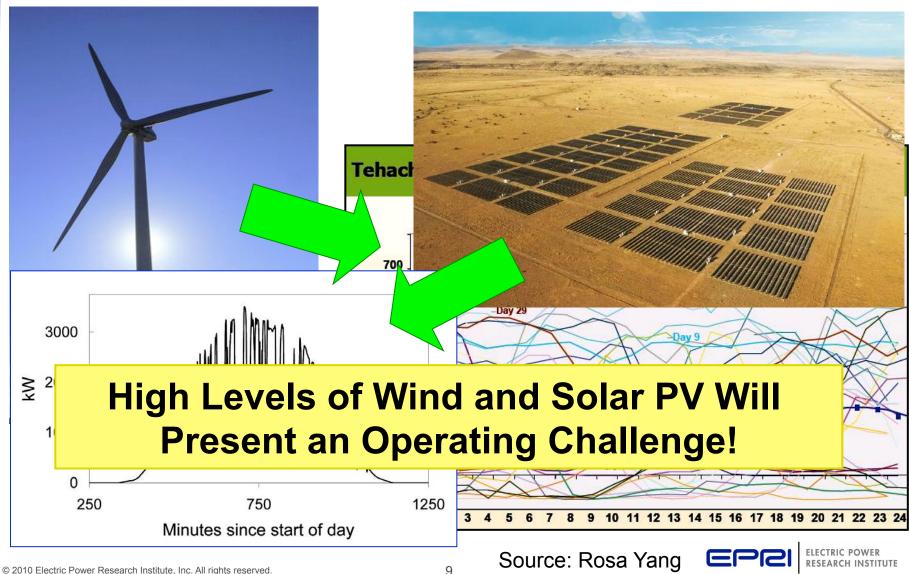
Source: DoE, Smart Grid Intro, 2008



Renewables in 2009

- 26% of global electricity capacity
- 18% of global electricity generation
- Developing countries have >50% of world's renewable capacity
- In both US & Europe, more than 50% of added capacity is renewable

Uncertainty of renewables



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Matching supply and demand

- Market as well as engineering challenges
- Slower timescale (minutes and up)
- Static power flow analysis

focus of this talk

Dynamic stability

- Engineering challenges
- Fast timescale (ms and up)
- Transient dynamics

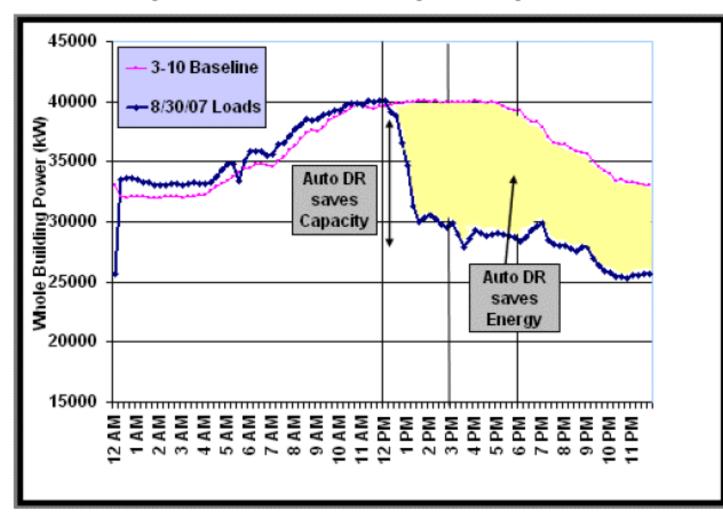


Mitigate uncertainty Matching deferrable loads to uncertain supply

Reduce peak US load factor ~55%

Automated Demand Response Saves Capacity and Energy

Electric load profile for PG&E participants on 8/30/2007



Source: Steven Chu, GridWeek 2009



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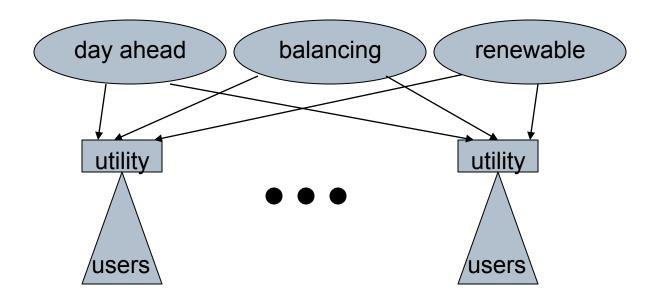
Preliminary results





Wholesale markets
 Day ahead, real-time balancing
 Renewable generation
 Non-dispatchable
 Demand response

Real-time control (through pricing)





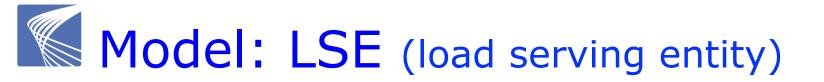
Each user has 1 appliance (wlog)

- Operates appliance with probability $\pi_i(t)$
- Attains utility $u_i(x_i(t))$ when consumes $x_i(t)$

$$\underline{x}_i(t) \leq x_i(t) \leq \overline{x}_i(t)$$

Demand at *t*:

$$D(t) := \sum_{i} \delta_{i} x_{i}(t) \qquad \delta_{i} = \begin{cases} 1 & \text{wp } \pi_{i}(t) \\ 0 & \text{wp } 1 - \pi_{i}(t) \end{cases}$$



Power procurement

Renewable power: $P_r(t)$, $c_r(P_r(t)) = 0$ Random variable, realized in real-time capacity energy

- Day-ahead power: $P_d(t)$, $c_d(P_d(t))$, $c_o(\Delta x(t))$ Control, decided a day ahead
- Real-time balancing power: $P_{b}(t), c_{b}(P_{b}(t))$

$$P_b(t) = D(t) - P_r(t) - P_d(t)$$

- Use as much renewable as possible
- Optimally provision day-ahead power
- Buy sufficient real-time power to balance demand



Simplifying assumptionNo network constraints

This talk: no time correlation Drop t in notations



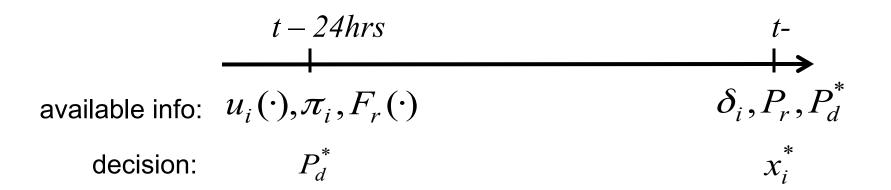
Day-ahead decision

How much power P_d should LSE buy from dayahead market?

Real-time decision (at *t*-)

How much x_i should users consume, given realization of wind power P_r and δ_i ?

How to compute these decisions distributively? How does closed-loop system behave ?





Real-time (at t-)

Given P_d and realizations of P_r, δ_i , choose optimal $x_i^* = x_i^* (P_d; P_r, \delta_i)$ to max social welfare, through DR

Day-ahead

Choose optimal P_d^* that maximizes expected optimal social welfare

available info:
$$u_i(\cdot), \pi_i, F_r(\cdot)$$

decision: P_d^*
 $t - 24hrs$
 $u_i(\cdot), \pi_i, F_r(\cdot)$
 δ_i, P_r, P_d^*



Focus: renewable integration

DR model

Preliminary results

- Distributed algorithms
- Effect of renewable on welfare





Supply cost

$$c(P_d, x) = c_d (P_d) + c_o (\Delta(x))_0^{P_d} + c_b (\Delta(x) - P_d)_+$$
$$\Delta(x) \coloneqq \sum_i \delta_i x_i - P_r \quad \longleftarrow \text{ excess demand}$$



 $c_d(P_d)$

Supply cost

$$c(P_{d}, x) = c_{d}(P_{d}) + c_{o}(\Delta(x))_{0}^{P_{d}} + c_{b}(\Delta(x) - P_{d})_{+}$$
$$\Delta(x) \coloneqq \sum_{i} \delta_{i} x_{i} - P_{r} \quad \text{excess demand}$$

 $c_d \left(\Delta(x) \right)_0^{P_d} \quad c_b \left(\Delta(x) - P_d \right)_+ \to \Delta(x)$



Supply cost

$$c(P_d, x) = c_d (P_d) + c_o (\Delta(x))_0^{P_d} + c_b (\Delta(x) - P_d)_+$$
$$\Delta(x) \coloneqq \sum_i \delta_i x_i - P_r \quad \longleftarrow \text{ excess demand}$$

Welfare function (random)

$$W(P_d, x) = \sum_i \delta_i u_i(x_i) - c(P_d, x)$$

user utility supply cost



Welfare function (random) $W(P_d, x) = \sum_i \delta_i u_i(x_i) - c(P_d, x)$

Optimal real-time demand response $\max_{x} W(P_{d}, x) \qquad \begin{array}{c} \text{given realization} \\ \text{of } P_{r}, \delta_{i} \end{array}$



Welfare function (random) $W(P_d, x) = \sum_i \delta_i u_i(x_i) - c(P_d, x)$

Optimal real-time demand response $x^*(P_d) := \arg \max_x W(P_d, x) \qquad \begin{array}{c} \text{given realization} \\ \text{of } P_r, \delta_i \end{array}$



Welfare function (random) $W(P_d, x) = \sum_i \delta_i u_i(x_i) - c(P_d, x)$

Optimal real-time demand response $x^*(P_d) := \arg \max_x W(P_d, x) \qquad \text{given realization} \ \text{of } P_r, \delta_i$

Optimal day-ahead procurement

$$P_d^* := \arg \max_{P_d} EW(P_d, x^*(P_d))$$

Overall problem: $\max_{P_d} E \max_{x} W(P_d, x)$



$$\max_{P_d} E \max_{x} W(P_d, x)$$
real-time DR

Active user *i* computes x_i^*

Optimal consumption

LSE computes

- Real-time price μ_b^*
 - Optimal day-ahead power to use y_o^*
- Optimal real-time balancing power y_b^{*}



Active user
$$i: x_i^{k+1} = \left(x_i^k + \gamma \left(u_i' \left(x_i^k\right) - \mu_b^k\right)\right)_{\underline{x}_i}^{\overline{x}_i}$$

inc if marginal utility > real-time price

LSE :

$$\mu_b^{k+1} = \left(\mu_b^k + \gamma \left(\Delta \left(x^k\right) - y_o^k - y_b^k\right)\right)_+$$

inc if total demand > total supply

- Decentralized
- Iterative computation at *t*-



Theorem: Algorithm 1

Socially optimal

- Converges to welfare-maximizing DR $x^* = x^* (P_d)$
- Real-time price aligns marginal cost of realtime power with individual marginal utility

$$\mu_{b}^{*} = c_{b}'(y_{b}^{*}) = u_{i}'(x_{i}^{*})$$

Incentive compatible

• x_i^* max *i*'s surplus given price μ_b^*



Theorem: Algorithm 1

Marginal costs, optimal day-ahead and balancing power consumed:

$$c_{b}'(y_{b}^{*}) = c_{o}'(y_{o}^{*}) + \mu_{o}^{*}$$

$$\downarrow$$

$$\mu_{o}^{*} = \frac{\partial W}{\partial P_{d}}(P_{d}^{*})$$

Algorithm 2 (day-ahead procurement)

Optimal day-ahead procurement

$$\max_{P_d} EW(P_d, x^*(P_d))$$

LSE:
$$P_d^{m+1} = \left(P_d^m + \gamma^m \left(\mu_o^m - c_d' \left(P_d^m\right)\right)\right)_+$$

calculated from Monte Carlo
simulation of Alg 1

Algorithm 2 (day-ahead procurement)

Optimal day-ahead procurement

$$\max_{P_d} EW(P_d, x^*(P_d))$$

LSE:
$$P_d^{m+1} = \left(P_d^m + \gamma^m \left(\mu_o^m - c_d' \left(P_d^m\right)\right)\right)_+$$

Given
$$\delta^m, P_r^m$$
: $\mu_o^m = \frac{\partial W}{\partial P_d} \left(P_d^m \right)$
 $\mu_b^m = \mu_o^m + c_o^{'} \left(y_o^m \right)$



Theorem

Algorithm 2 converges a.s. to optimal P_d^* for appropriate stepsize γ^k



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Renewable power:

$$P_r(a,b) := a \cdot \mu_r + b \cdot V_r$$

$$\uparrow \qquad \uparrow$$

$$mean \quad \text{zero-mean RV}$$

Optimal welfare

$$W(P_r(a,b)) := \max_{P_d} E \max_{x} W(P_d,x)$$



$$P_r(a,b) \coloneqq a \cdot \mu_r + b \cdot V_r$$

$$W(P_r(a,b)) := \max_{P_d} E \max_{x} W(P_d,x)$$

Theorem

Cost increases in var of P_r $W(P_r(a,b))$ increases in a, decreases in b $W(P_r(s,s))$ increases in s but no ramp constraint!