A Mean Field Games Formulation of Network Based Auction Dynamics

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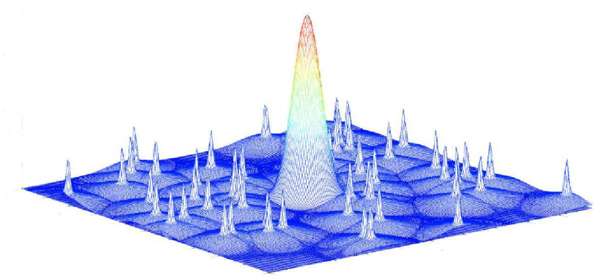
Basic Ideas of Mean Field Games

Part 1 – CDMA Power Control

Base Station & Individual Agents



Part 1 – CDMA Power Control

- Lognormal channel attenuation: $1 \le i \le N$

  i_{th} channel: $dx_i = -a(x_i + b)dt + \sigma dw_i, \qquad 1 \le i \le N$

  Transmitted power $=$ channel attenuation $\times$ power

  $= e^{x_i(t)} p_i(t)$

  (Charalambous, Menemenlis; 1999)

  Signal to interference ratio (Agent i) at the base station

  $= e^{x_i} p_i / \left[(\beta/N) \sum_{j \neq i}^{N} e^{x_j} p_j + \eta \right]$

- How to optimize all the individual SIR's?
  - Self defeating for everyone to increase their power
  - Humans display the "Cocktail Party Effect": Tune hearing to frequency of friend's voice (E. Colin Cherry)

Part 1 – CDMA Power Control

- Can maximize $\sum_{i=1}^{N} SIR_i$ with centralized control.
(HCM, 2004)
- Since centralized control is not feasible for complex systems, how can such systems be optimized using decentralized control?
- Idea: Use large population properties of the system together with basic notions of game theory.
- Massive game theoretic control systems: Large ensembles of partially regulated competing agents
- Fundamental issue: The relation between the actions of each individual agent and the resulting mass behavior

Part 2 – Basic LQG Game Problem

Individual Agent's Dynamics:

$$dx_i = (a_i x_i + bu_i)dt + \sigma_i dw_i, \quad 1 \le i \le N.$$

(scalar case only for simplicity of notation)

- x_i: state of the ith agent
- u_i: control
- w_i: disturbance (standard Wiener process)
- N: population size

Part 2 – Basic LQG Game Problem

Individual Agent's Cost:

$$J_i(u_i, \nu) \triangleq E \int_0^\infty e^{-\rho t}[(x_i - \nu)^2 + r u_i^2] dt$$

Basic case: $\nu \triangleq \gamma.(\frac{1}{N} \sum_{k \neq i}^N x_k + \eta)$

Main features:

- Agents are coupled via their costs
- Tracked process ν:
  - (i) stochastic
  - (ii) depends on other agents' control laws
  - (iii) not feasible for x_i to track all x_k trajectories for large N

Part 2 – Large Popn. Models with Game Theory Features

- Economic models: Cournot-Nash equilibria (Lambson)
- Advertising competition: game models (Erickson)
- Wireless network res. alloc.: (Alpcan et al., Altman, HCM)
- Admission control in communication networks: (Ma, MC)
- Public health: voluntary vaccination games (Bauch & Earn)
- Biology: stochastic PDE swarming models (Bertozzi et al.)
- Sociology: urban economics (Brock and Durlauf et al.)
- Renewable Energy: Charging control of of PEVs (Ma et al.)

Part 2 – Preliminary Optimal LQG Tracking

LQG Tracking: Take x^* (bounded continuous) for scalar model:

$$dx_i = a_i x_i dt + b u_i dt + \sigma_i dw_i$$

$$J_i(u_i, x^*) = E\int_0^\infty e^{-\rho t}[(x_i - x^*)^2 + r u_i^2]dt$$

Riccati Equation: $\rho\Pi_i = 2a_i\Pi_i - \frac{b^2}{r}\Pi_i^2 + 1, \quad \Pi_i > 0$

Set $\beta_1 = -a_i + \frac{b^2}{r}\Pi_i$, $\beta_2 = -a_i + \frac{b^2}{r}\Pi_i + \rho$, and assume $\beta_1 > 0$

Mass Offset Control: $\rho s_i = \frac{ds_i}{dt} + a_i s_i - \frac{b^2}{r}\Pi_i s_i - x^*.$

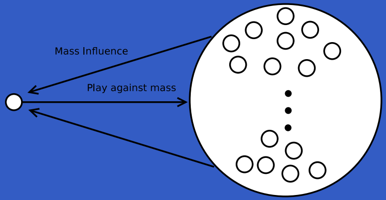
Optimal Tracking Control: $u_i = -\frac{b}{r}(\Pi_i x_i + s_i)$

- Boundedness condition on x^* implies existence of unique solution s_i.

Part 2 – Key Intuition

When the tracked signal is replaced by the deterministic mean state of the mass of agents:

Agent's feedback = feedback of agent's local stochastic state



+

feedback of deterministic mass offset

Think Globally, Act Locally (Geddes, Alinsky, Rudie-Wonham)

The Fundamental NCE Equation System

Continuum of Systems: $a \in \mathcal{A}$; common b for simplicity

$$\rho s_a = \frac{ds_a}{dt} + a s_a - \frac{b^2}{r}\Pi_a s_a - x^*$$

$$\frac{d\overline{x}_a}{dt} = (a - \frac{b^2}{r}\Pi_a)\overline{x}_a - \frac{b^2}{r}s_a,$$

$$\overline{x}(t) = \int_{\mathcal{A}} \overline{x}_a(t) dF(a),$$

$$x^*(t) = \gamma(\overline{x}(t) + \eta) \qquad t \geq 0$$

$$\textbf{Riccati Equation :} \quad \rho \Pi_a = 2a\Pi_a - \frac{b^2}{r}\Pi_a^2 + 1, \quad \Pi_a > 0$$

- Individual control action $u_a = -\frac{b}{r}(\Pi_a x_a + s_a)$ is optimal w.r.t tracked x^*.
- Does there exist a solution $(\overline{x}_a, s_a, x^*; a \in \mathcal{A})$?
  Yes: Fixed Point Theorem

Part 2 – NCE Feedback Control

Proposed MF Solution to the Large Population LQG Game Problem
The Finite System of N Agents with Dynamics:

$$dx_i = a_i x_i dt + b u_i dt + \sigma_i dw_i, \qquad 1 \le i \le N, \qquad t \ge 0$$

Let $u_{-i} \triangleq (u_1, \cdots, u_{i-1}, u_{i+1}, \cdots, u_N)$; then the individual cost

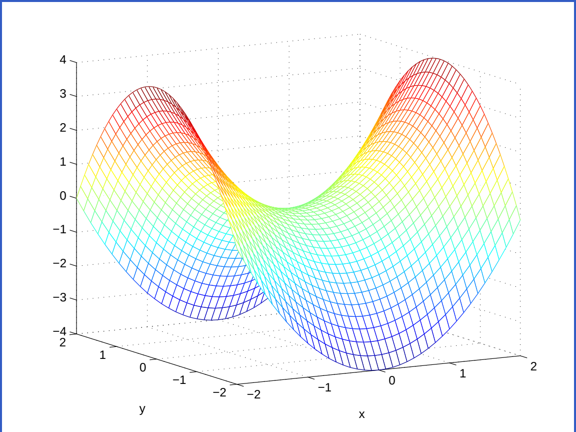
$$J_i(u_i, u_{-i}) \triangleq E \int_0^\infty e^{-\rho t} \{[x_i - \gamma(\frac{1}{N} \sum_{k \neq i}^{N} x_k + \eta)]^2 + r u_i^2\} dt$$

Algorithm: For ith agent with parameter (a_i, b) compute:

- x^* using NCE Equation System
- $$\begin{cases} \rho \Pi_i = 2 a_i \Pi_i - \frac{b^2}{r} \Pi_i^2 + 1 \\ \rho s_i = \frac{ds_i}{dt} + a_i s_i - \frac{b^2}{r} \Pi_i s_i - x^* \\ u_i = -\frac{b}{r}(\Pi_i x_i + s_i) \end{cases}$$

Part 2 – Saddle Point Nash Equilibrium

- Agent y is a maximizer
- Agent x is a minimizer



Part 2 – Nash Equilibrium

The Information Pattern:

$\mathcal{F}_i \triangleq \sigma(x_i(\tau); \tau \leq t)$ $\mathcal{F}^N \triangleq \sigma(x_j(\tau); \tau \leq t, 1 \leq j \leq N)$

$\mathcal{F}_i$ adapted control: $\mathcal{U}_{loc,i}$ $\mathcal{F}^N$ adapted control: $\mathcal{U}$

The Equilibria:

The set of controls $\mathcal{U}^0 = \{u_i^0;\, u_i^0$ adapted to $\mathcal{U}_{loc,i},\ 1 \leq i \leq N\}$ generates a *Nash Equilibrium* w.r.t. the costs $\{J_i; 1 \leq i \leq N\}$ if, for each i,

$$J_i(u_i^0, u_{-i}^0) = \inf_{u_i \in \mathcal{U}} J_i(u_i, u_{-i}^0)$$

Part 2 – ϵ-Nash Equilibrium

ϵ-Nash Equilibria:

Given $\varepsilon > 0$, the set of controls $\mathcal{U}^0 = \{u_i^0; 1 \leq i \leq N\}$ generates an *ε-Nash Equilibrium* w.r.t. the costs $\{J_i; 1 \leq i \leq N\}$ if, for each i,

$$J_i(u_i^0, u_{-i}^0) - \varepsilon \leq \inf_{u_i \in \mathcal{U}} J_i(u_i, u_{-i}^0) \leq J_i(u_i^0, u_{-i}^0)$$

Part 2 – NCE Control: First Main Result

Theorem 1: (MH, PEC, RPM, 2003)

Subject to technical conditions, the NCE Equations have a unique solution for which the NCE Control Algorithm generates a set of controls $\mathcal{U}_{nce}^N = \{u_i^0; 1 \le i \le N\}, \ 1 \le N < \infty$, where

$$u_i^0 = -\frac{b}{r}(\Pi_i x_i + s_i)$$

which are s.t.

(i) All agent systems $S(A_i), \ 1 \le i \le N,$ are second order stable.

(ii) $\{\mathcal{U}_{nce}^N; 1 \le N < \infty\}$ yields an ε-Nash equilibrium for all ε, i.e. $\forall \varepsilon > 0 \ \exists N(\varepsilon)$ s.t. $\forall N \ge N(\varepsilon)$

$$J_i(u_i^0, u_{-i}^0) - \varepsilon \le \inf_{u_i \in \mathcal{U}} J_i(u_i, u_{-i}^0) \le J_i(u_i^0, u_{-i}^0),$$

where $u_i \in \mathcal{U}$ is adapted to $\mathcal{F}^N$.

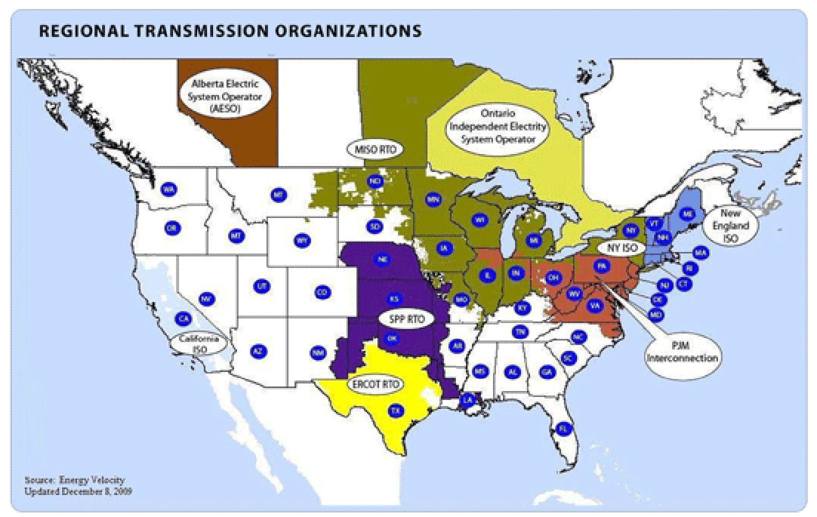
□

Network Based Auctions and Applications of MFG

Part 3 – Network Based Auction: Overview

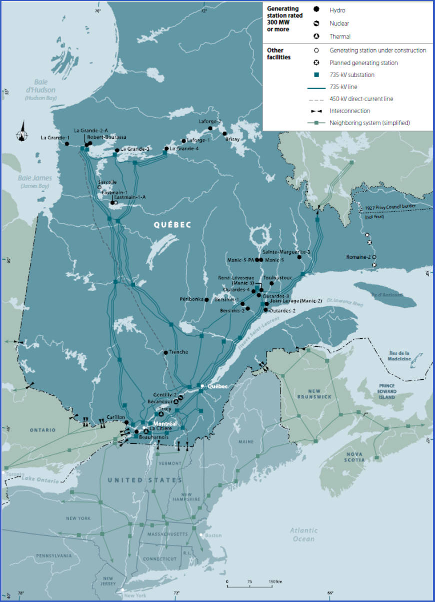
- Game theoretic methods for market pricing and resource allocation on distributed networks
    - Two-level network structure
    - Lower level: quantized progressive second price auctions with fixed local quantities
    - Higher level: cooperative consensus allocation of local quantities
- Convergence and efficiency analysis of network based auctions
- Applications of Mean Field Game to auctions and networks

Part 3 – ISO / RTO



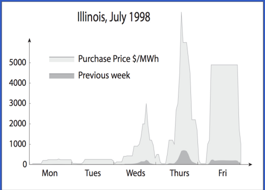
Part 3 – Hydro-Québec

- 60 hydroelectric generating stations
- 36,971 MW installed capacity
- 175 TW storage capacity
- 579 dams, 97 control structures

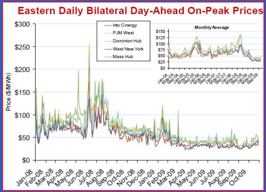


www.hydroforthefuture.com

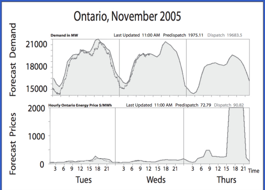
Part 3 – Worldwide Examples of Extreme Price Volatility



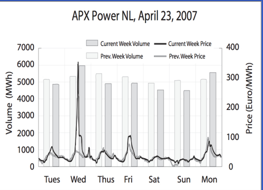
Illinois [1]



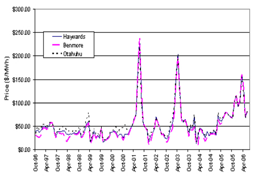
East US [2]



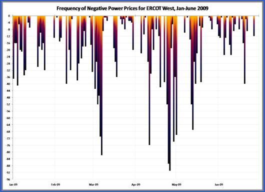
Ontario [1]



The Netherlands [1]



New Zealand [3]



West Texas [4]

[1] Cho & Meyn, 2010 [2] http://www.ferc.gov [3] http://www.treasury.govt.nz [4] Giberson, 2008

Part 3 – Quantized PSP Auctions (Jia & Caines 2011)

- A non-cooperative game;
- N buyer agents bid for a divisible resource C;
- Given a finite price set B_p^0, each buyer agent BA_i makes a *quantized bid*: $s_i = (p_i, q_i) = (\text{price, quantity})$, $p_i \in B_p^0$;
- A *bid profile* is $s = (s_1, \cdots, s_N)$;
- $\theta_i : \mathcal{R}^+ \to \mathcal{R}^+$, is the *valuation function*, and $\theta_i^{'}$ is the (decreasing) *demand function*;
- A *market price function* (MPF) for BA_i is

$$P_i(z, s_{-i}) = \inf \left\{ y \geq 0 : C - \sum_{p_k > y, k \neq i} q_k \geq z \right\}.$$

Objective: Design a market mechanism (i.e., assignment of allocations) and find a bidding rule for each agent which individually maximizes its utility function and which leads to a Nash equilibria and which is socially efficient (i.e. max sum individual utilities).

Part 3 – PSP Mechanism (celebrated VCG mechanism)

The PSP *allocation rule* and *cost function* are defined as:

$$a_i(s) = a_i((p_i, q_i), s_{-i}) = \min\{q_i, \frac{q_i}{\sum_{k:p_k=p_i} q_k} Q_i(p_i, s_{-i})\},$$

(reasonable: MPF constrained allocation)

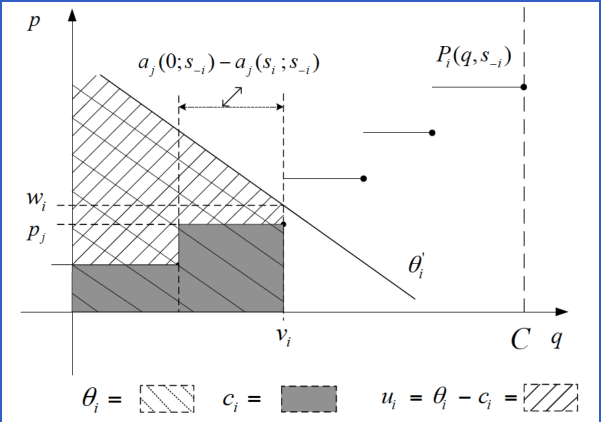
$$c_i(s) = \sum_{j \neq i} p_j \left[a_j((0,0), s_{-i}) - a_j(s_i, s_{-i})\right],$$

(reasonable: corresponding to opportunity costs)

where $Q_i(y, s_{-i})$ is the available quantity at price y given s_{-i}.

Then BA_i's *utility function* $u_i(s) = \theta_i(a_i(s)) - c_i(s)$.

Part 3 – Best Reply



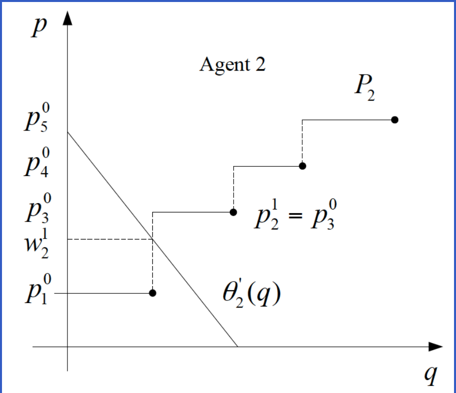
Given s_{-i} and *elastic* θ_i', utility maximum implies the best (bid) reply,

$$v_i = \left[\sup\left\{q \geq 0 : \theta_i'(q) > P_i(q, s_{-i})\right\}\right]^+, w_i = \theta_i'(v_i) \in \mathcal{R}^+.$$

Part 3 – Quantized Strategies

A generic buyer, e.g., Agent 2:

- Applies the same utility function and allocation rule as PSP.
- Makes the quantized price and quantity bid: $p_i^k \in B_p^0, q_i^k = \theta_i^{'-1}(p_i^k), 1 \leq i \leq N, k \geq 0$, where there is no bid fee.
- Bids are made synchronously.



Part 3 – Quantized PSP State-Space Dynamical System

$$P_i^{k+1}(q, s_{-i}^k) = \arg\inf_{p \geq 0} \left\{ C \geq q + \sum_{p_j^k > p, j \neq i} q_j^k \right\},$$

$$v_i^{k+1} = \sup \left\{ q \geq 0 : \theta_i'(q) > P_i^{k+1}(q, s_{-i}^k) \right\},$$

(best quantity reply given s_{-i}^k)

$$(p_i^{k+1}, q_i^{k+1}) = \left(T \left(v_i^{k+1}, s^k, B_p^0 \right), D_i(p_i^{k+1}) \right), \quad \forall 1 \leq i \leq N.$$

(quantized strategy)

Note:

- $p_i^k \in B_p^0$, $D_i = \theta_i'^{-1}$, and T is a quantization operation of v_i.
- (p_i^k, q_i^k) is *γ-best reply and truth-telling*: γ depending on B_p^0.

Part 3 – Convergence of Q-PSP

Theorem 2: (PJ&PEC 2010) Subject to some mild assumptions, the dynamical Q-PSP system converges in at most k^* iterations to the unique price p^*, which satisfies

$$p^* = \min\{p \in B_p^0 : \sum_{1 \le i \le N} D_i(p) \le C\}$$

where k^* satisfies

$$k^* = |\{p \in B_p^0 : \sum_{1 \le i \le N} D_i(p) > C\}| + 1.$$

Proof:

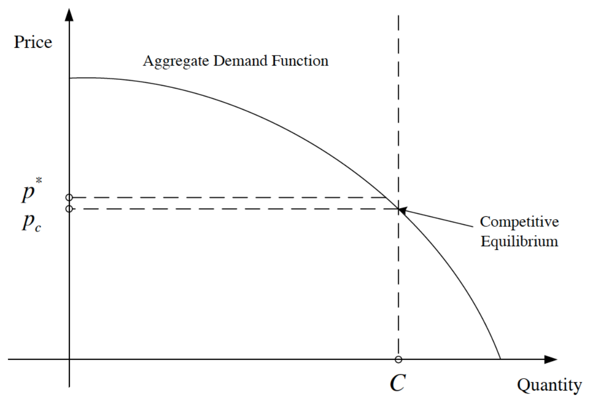
- $\min\{p_i^k\}$ is monotonically decreasing.
- $\min\{p_i^k\} = \max\{p_i^k\}$ in the limit.

$\sum_i D_i(\cdot)$ is called the (inverse) aggregate demand function. □

Part 3 – Properties of the Limit

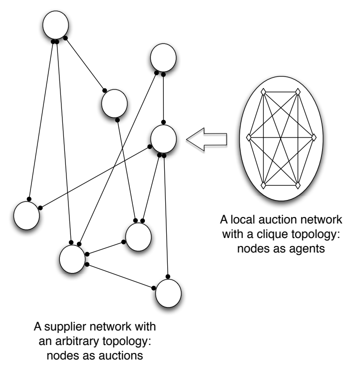
- The limit bidding profile s^* is a $\gamma(B_p^0)$-*NE*.
- The limit allocation is *efficient* (i.e., $\max \sum_i \theta_i$) up to $\sqrt{\gamma(B_p^0)}$ under mild assumptions on demand functions.
- k^* is independent of the number of buyer agents.
- p^* and k^* are independent of the initial bidding profile.

Part 3 – Approximation of Competitive Equilibrium



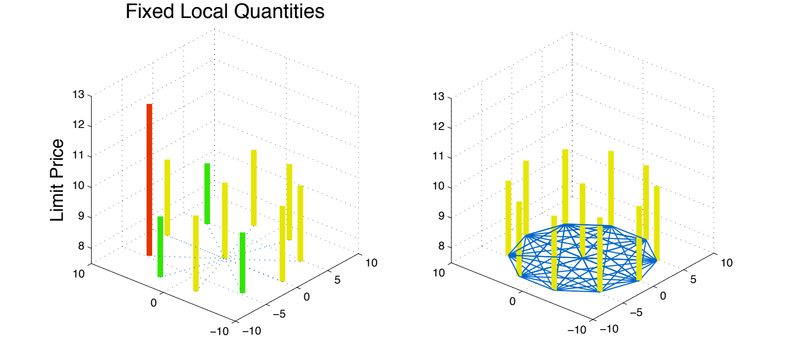
p_c is called *market clearing price* and it can be shown to correspond to an efficient allocation under mild assumptions on demand functions. $p_c > \max\{p \in B_p^0, p < p^*\}$.

Part 3 – Two-level Network-Based Auction (NBA)



- M Vertices on the higher level network with an arbitrary topology $G = (V, E)$ are suppliers.
- Vertices on the lower level networks with a clique topology represent buyers.
- Each lower level network associated with one supplier is a local Q-PSP auction G_h.
- $C = \sum_{h=1}^{M} C_h$ is fixed and all networks are connected.

Part 3 – Local Limit Prices Vs. Global Limit Price



Distributed Auctions

Single Auction

Part 3 – Consensus Analysis of Local Quantities

- Unbalanced fixed local quantities prevent a globally efficient allocation being achieved.

- Local quantities are adjusted cooperatively based on their neighbors' information (quantities and quantized limit prices):

  Quantity re-allocation algorithm

$$C_h(k+1) - C_h(k) = \sum_{j \in N_h} \Phi_{hj}(C_j(k), C_h(k), p_j^*(k), p_h^*(k)), \quad 1 \le h \le M.$$

  (Superscript * denotes quantization in the following context.)

- The time scale of the higher level network is significantly larger than that in local auctions.

Part 3 – Passivity Condition

Lemma: For any local auction G_h, the corresponding limit price function $p_h^*(C)$, for a given quantity C, satisfies the *passivity property*:

$$(p_h^*(C_1) - p_h^*(C_2))(C_1 - C_2) \leq 0, \quad \forall\, 1 \leq h \leq M. \qquad \square$$

This is a consequence of the decreasing property of the demand functions and the nature of limit prices of Q-PSP auctions.

Part 3 – Convergence of Two-level NBA

Theorem 3: (PJ&PEC 2011) Consider a (two-level) network-based Q-PSP auction associated with a connected higher level network topology and the quantity re-allocation algorithm:

$$\Phi_{hj}(C_j, C_h) = -\alpha \cdot (p_j^*(C_j) - p_h^*(C_h)), \quad \forall\, 1 \leq h \leq M.$$

where quantized $p_h^*(\cdot) \in B_p^0$ for any $1 \leq h \leq M$. Then there exist a sufficiently small $\alpha > 0$ and limit quantities $\{C_h^\infty, 1 \leq h \leq M\}$ with $\sum_h C_h^\infty = C$, such that, for any initial condition:

$$\{C(k), p^*(k)\} \text{ converges to } [C_h^\infty, p_g^*]_{1 \leq h \leq M},$$

where $p_h^*(C_h^\infty) = p_g^*$ for all h. □

Part 3 – Convergence of Two-level NBA: Proof

Proof:

- The weighted consensus dynamics is formulated such that:

$$C(k+1) = C(k) + \alpha L p^*(C(k))$$
$$\Rightarrow \qquad p(k+1) = p(k) - \alpha\beta(k) L p^*(k), \ \forall\, k \geq 0,$$

  where $\beta(k) > 0$ depends upon the aggregate local demand functions.
  (It is noted p is continuously valued and calculated from C and β, and p^* is the quantized local limit price vector.)

- The consensus to a unique price p_g^* is achieved since
  - all the Perron matrices generated in the algorithm are SIA (stochastic, indecomposable and aperiodic), and
  - all positive entries of the Perron matrices are lower-bounded □

Note: p_g^* is the *quantized market clearing price* for the entire network.

Part 3 – Extension with Continuous Pricing

Theorem 4: (PJ&PEC 2011) Consider a (two-level) network-based Q-PSP auction. Assume the higher level network is connected, the local prices are permitted to take continuous real values, and

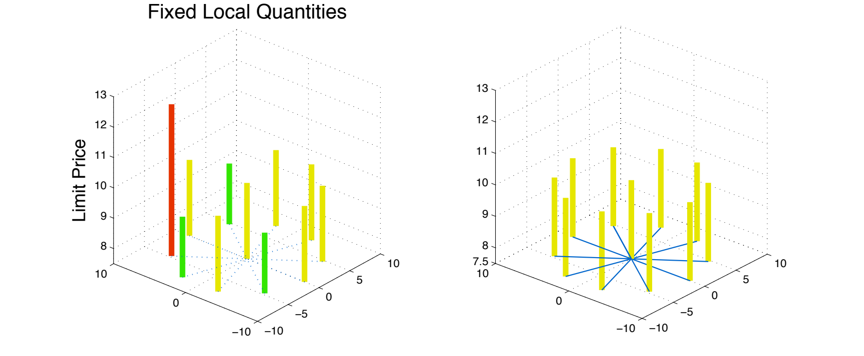
$$\Phi_{hj}(C_j, C_h) = -\alpha \cdot (p_j(C_j) - p_h(C_h)), \quad \forall\, 1 \leq h \leq M,$$

then there exist a unique set $\{C_h^\infty, 1 \leq h \leq M\}$ with $\sum_h C_h^\infty = C$ and a unique price p_g, s. t., for any initial condition,

$\{C(k), p(k)\}$ converges geometrically to $[C_h^\infty, p_g]_{1 \leq h \leq M}$. □

Note: p_g is the *Global Market Clearing Price (GMCP)* (parallel to p_c in a single auction).

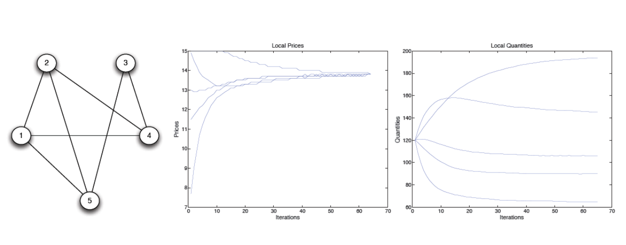
Part 3 – Effect of Local Quantity Consensus

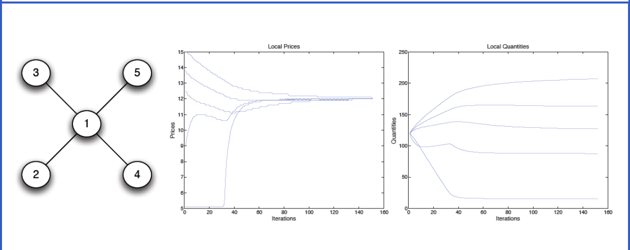


No Connection

Star Network

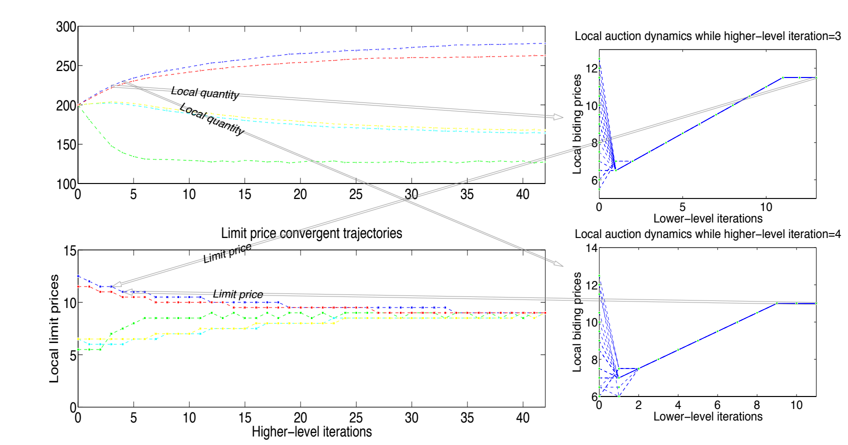
Part 3 – Numerical Examples





The convergence of quantized NBAs with different network topology.

Numerical Examples Cont.



Two level dynamics.

Part 3 – Static MF Strategies for Quantized Auctions

- If s_{-i} is not completely known to Buyer Agent BA_i, the quantized strategy is not feasible directly.
- Mean Field Framework:
  - Each buyer agent is assumed to have a statistical distribution on the demand functions of the population.
  - Apply quantized strategies for an infinite population at each time instant.
  - The price distribution converges to a delta unit mass function on p^*, as each buyer agent can solve for it instantaneously from the expected aggregate demand function and the total capacity.
  - Each buyer agent in the finite population case uses the infinite population best reply w.r.t p^*.

Part 3 – MF application on NBA: Motivations

- Prior info + MFG: convergence to the limit for very large population, independent of network topologies.
- If network connectivity temporarily breaks down the consensus theory cannot be used and MFG is an excellent substitute.

Part 3 – Cont. Time NBA with MF Strategy

- Assumption 1: In the lower level auctions limit prices are achieved instantaneously w.r.t. the higher level dynamics.
- A continuous time (large population) stochastic NBA problem is formulated as a dynamic game with:
  - The stochastic dynamics for each supplier SA_h:

    $$dC_h(t) = u_h(t)dt + \sigma dw_h(t),\ 1 \leq h \leq M, t \geq 0.$$

    C_h: state of supplier SA_h, u_h: control input, $\{w_h\}$: independent Wiener processes.
  - Since $dC_h(t) = -dp_h(t)/\beta_h(t)$ (from the aggregate local demand functions), for simplicity of analysis:

    $$dp_h(t) = -\beta_h(t)(u_h(t)dt + \sigma dw_h(t)),\ 1 \leq h \leq M, t \geq 0.$$

Part 3 – Empirical Initial State Distribution

Assumption 2: The initial state distribution function F satisfies $\int_A dF(\xi) = 1$ where A is a compact set containing all initial local limit prices. Denote the empirical distribution function for M suppliers

$$F^{(M)}(x) := \frac{1}{M}\sum_{h=1}^{M} 1_{p_h(0)\leq x}.$$

It is assumed that $\{F^{(M)}, M \geq 1\}$ converges to F weakly: for any bounded and continuous function ϕ defined on $\mathcal{R}$,

$$\lim_{M\to\infty}\int \phi(x)dF^{(M)}(x) = \int \phi(x)dF(x),$$

Part 3 – Cost, Mass Behavior and MF Strategy

- Individual (supplier) *long run average* cost is:

$$J_h = \lim_{T\to\infty} \inf \frac{1}{T} \int_0^T ([p_h(C_h) - \frac{\sum_{k\neq h}^{M} p_h(C_h)}{M-1}]^2 + r u_h^2) dt, \; r > 0.$$

- Given a distribution F of initial states , the *MF equation system* for infinite population is

$$\begin{aligned} ds_\xi(t)/dt &= s_\xi(t)/\sqrt{r} + p^*(t), \\ d\bar{p}_\xi(t)/dt &= \beta_\xi \cdot (\bar{p}_\xi(t)/\sqrt{r} + s_\xi(t)/r), \\ p^*(t) &= \int \bar{p}_\xi(t) dF. \end{aligned}$$

- *MF strategy* is $u_\xi(t) = \beta_\xi \cdot (p_\xi(t)/\sqrt{r} + s_\xi(t)/r)$.

Part 3 – MF Strategy and Closed-loop MF System

- MF equation system has a unique solution $p^*(t)$ in infinite population, $s(p^*(t))$ is then available.
- $\lim_{t\to\infty} p^*(t) = p_g$ (GMCP), i.e., $\lim_{t\to\infty} \int \bar{p}_\xi(t)dF = p_g$.
- Then each supplier SA_h applies the infinite population MF strategy in the finite population case:

$$u_h^o(t) = \beta_h \cdot (p_h^o(t)/\sqrt{r} + s_h(p^*(t))/r).$$

- The resulting closed-loop dynamics is:

$$dp_h^o(t) = \beta_h \cdot (p_h^o(t)/\sqrt{r} + s_h(p^*(t))/r)dt + \sigma dw_h(t).$$

Part 3 – MF Consensus

Theorem 5: (PJ&PEC 2012) Subject to the instantaneous convergence assumption on the lower level dynamics and the empirical initial state distribution assumption, if all suppliers in the higher level network apply MF strategies:

$$u_h^o(t) = \beta_h \cdot (p_h^o(t)/\sqrt{r} + s_h(p^*(t))/r),$$

then a mean consensus is asymptotically reached almost surely and

$$\lim_{M\to\infty} \frac{1}{M}\sum_{h=1}^{M} p_h^o(t) = p^*(t), \text{ a.s. } dF,$$

where $\lim_{t\to\infty} |\bar{p}_h^o(t) - p_g| = 0$ for all $1 \le h \le M$, which corresponds to an ε-Nash equilibrium. □

ε is the difference between the initial state average of the finite population and the expected initial state of an infinite population.

Part 3 – Challenges for MFG Limits of Network Consensus Algorithms

- Prior info + MFG: convergence to the limit for very large population, independent of network topologies.
- If network connectivity temporarily breaks down the consensus theory cannot be used and MFG is an excellent substitute.
- If the prior data on "current initial conditions" gets updated (by observation or adaptation) then we can recompute the MFG solution. But an "optimal" finite time theory is still needed unless we go to full stochastic adaptive control theory solution. (Kizilake and PEC).
- The controlled (i.e not in response to network breakdown) mix of MFG and Consensus (optimal) is still to be worked out.
- The higher level substitution of an MFG algorithm does not need to be an competitive NCE algorithm but can be a cooperative (SCE) , with very similar results.

Summary

- MFG is a theory for solving a class of decentralized decision-making problems with many competing agents. Auctions are an example of these problems.
- Quantized PSP auction is developed for fast convergence and realistic modelling.
- Two-level NBA is designed for Q-PSP with incomplete bidding information. A consensus on the local limit prices is achieved by the NBA algorithm, which corresponds to a quantized efficient quantity allocation.
- Fragile networks and expensive communication lead to MFG at the upper level which yields a mean consensus and an ε-NE, which corresponds to a near-efficient allocation