A networked control strategy for reactive power compensation in a smart power distribution network

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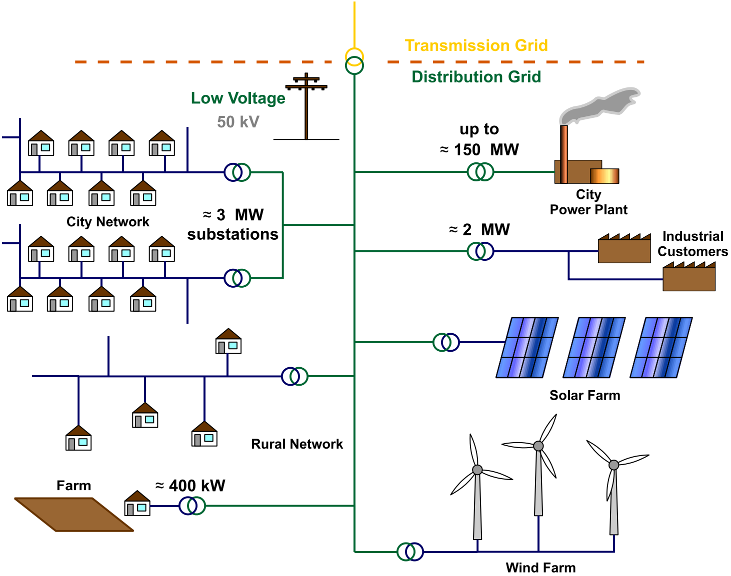
REACTIVE POWER COMPENSATION

Power distribution networks



Smart power distribution grid

Smart microgenerators

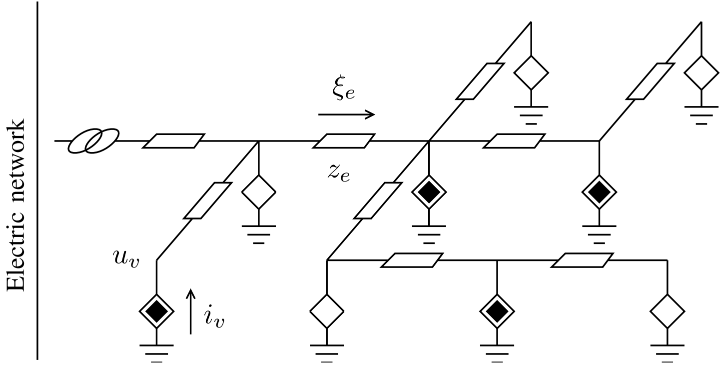
We consider a portion of the electrical power distribution network populated by a number of microgeneration devices (solar panels, ...), each of them equipped with sensing and communication capabilities.

The power electronics of these microgenerators can be exploited for providing useful ancillary services.

We focus on the problem of optimal reactive power compensation for the minimization of distribution losses.

Power distribution grid

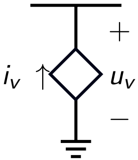
We assume that voltages and currents are sinusoidal signals, at the same frequency, and thus described by their amplitude and phase.



Reactive power

Reactive power flows

Whenever a device in the grid injects (is supplied with) a current that is out of phase with the voltage, we have injection (delivery) of reactive power.



Adopting the phasorial notation for voltages and currents, we define the complex power

$$s_v = p_v + j q_v := u_v \bar{i}_v$$

Reactive power “facts”

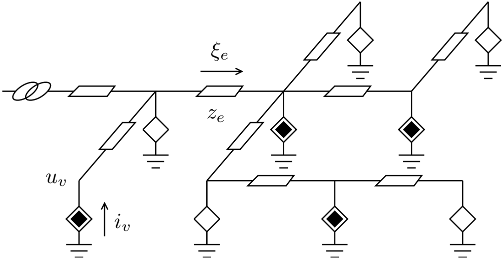
- Loads in the microgrid require reactive power
- reactive power can be obtained from the transmission grid or produced by the microgenerators in the grid
- producing reactive power has no fuel cost
- larger flows of reactive power correspond to quadratically larger power losses on the cables.

Optimal reactive power compensation problem

Injecting reactive power in the grid as close as possible to the loads that need it, in order to minimize power distribution losses.

MICROGRID MODEL

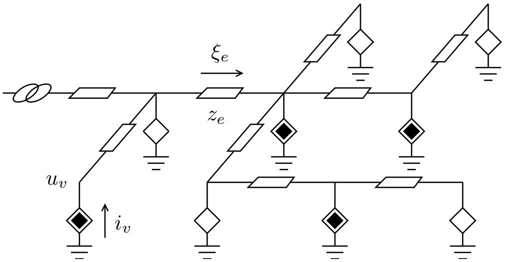
Graph model



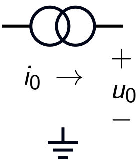
Nodes of the graph represent loads (in white) that cannot be controlled, and microgenerators (in black) which can be commanded, can sense the grid, and can communicate.

Nodes are connected by a tree $\mathcal{T}$, representing the electrical connection (power lines) among them.

Graph model



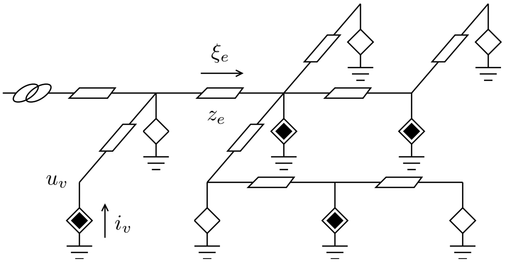
Node 0

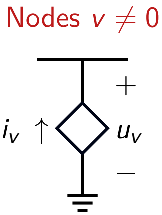


Node 0 represents the point of connection of the microgrid to the transmission grid.
Its voltage u_0 corresponds in amplitude to the nominal voltage U_N of the microgrid:

$$u_0 = U_N e^{j\phi_0}.$$

Graph model



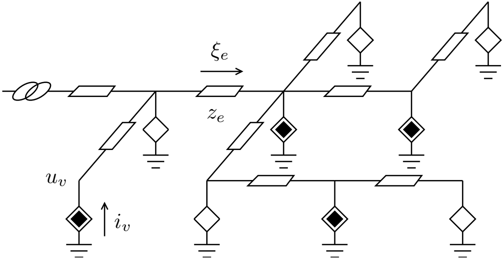


Node voltage u_v and node current i_v satisfy

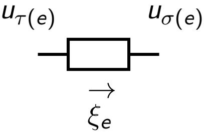
$$u_v \bar{i}_v = s_v$$

for microgenerators and loads (can be extended to exponential / ZIP model).

Graph model



Edges e



Voltage drop $u_{\tau(e)} - u_{\sigma(e)}$ and the current ξ_e flowing on the edge e satisfy

$$u_{\tau(e)} - u_{\sigma(e)} = z_e \xi_e$$

where z_e is the impedance of the power line e.

Microgrid nonlinear equations

The voltages u_v and the currents i_v of the microgrid are therefore implicitely defined by the system of nonlinear equations

$$\begin{cases} Lu = i \\ u_v \bar{i}_v = s_v & \qquad v \neq 0 \\ u_0 = U_N e^{j\phi_0}, \end{cases}$$

where L is the weighted Laplacian of the graph

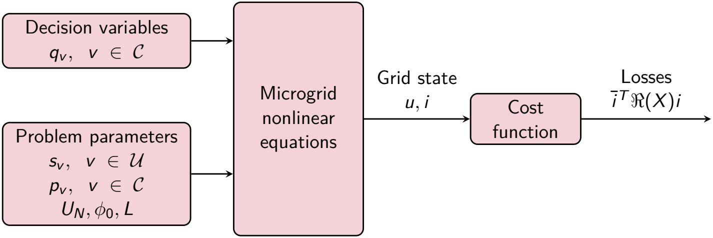
$$L = A^T Z^{-1} A$$

and A is the incidence matrix of the graph.

OPTIMIZATION PROBLEM

Optimization problem

The optimization problem consists in deciding the reactive power injection at the microgenerators that minimizes power distribution losses.



In order to design an algorithm we need an explicit expression for the grid state as a function of the decision variables.

Explicit grid solution

Approximate solution

We constructed the Taylor expansion of the system state for large nominal voltage U_N.

$$i_v(U_N) = \frac{\bar{s}_v}{U_N} + \frac{c_v(U_N)}{U_N^2}$$

$$u_v(U_N) = U_N + \frac{[X\bar{s}]_v}{U_N} + \frac{d_v(U_N)}{U_N^2}.$$

This model extends the DC power flow model, by relaxing the assumption of zero losses (i.e. purely inductive lines).

Approximate problem

The approximate solution of the grid equations allows us to rewrite the cost function (losses) as a quadratic function of the decision variables.

$$J = \frac{1}{U_N^2} p^T \Re(X)\, p + \frac{1}{U_N^2} q^T \Re(X)\, q + \frac{1}{U_N^3} \tilde{J}(p, q, U_N)$$

where $\tilde{J}$ is bounded for large U_N, and q satisfies $1^T q = 0$.

Quadratic cost function

We approximated the original problem as a convex quadratic optimization problem subject to a linear equality constraint.

DISTRIBUTED ALGORITHM

Motivation for a distributed algorithm

Implementing a centralized solver for the quadratic (linearly constrained) optimization problem is impossible:

- complete knowledge of the system parameters

  $$L, \quad \{p_v,\, v \in \mathcal{C}\}, \quad \{s_v,\, v \in \mathcal{U}\}$$

  and state

  $$\{q_v,\, v \in \mathcal{C}\}$$

  is required
- coordination and communication among all nodes $\mathcal{U} \cup \mathcal{C}$ is required
- compensators
  - are in large number
  - can connect and disconnect
  - have limited communication capabilities.

Distributed architecture

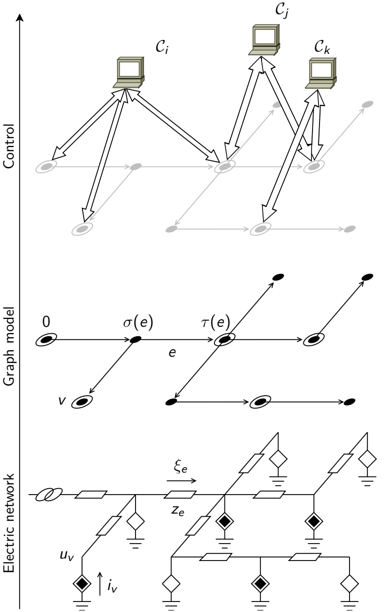
Consider the family of subsets of $\mathcal{C}$

$$\{\mathcal{C}_1, \ldots, \mathcal{C}_\ell\}$$

such that $\bigcup_{i=1}^{\ell} \mathcal{C}_i = \mathcal{C}$.

Let each cluster be managed by an intelligent unit (possibly, one of the compensators), which

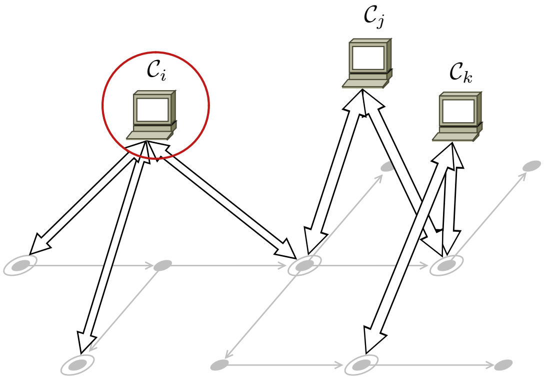
- knows the relative position of the compensators
- collects data from the compensators
- processes the collected data
- commands the compensators.



Iterative algorithm

At (possibly uneven) time step

1) a cluster $\mathcal{C}_i$ activates;
2) the supervisor of $\mathcal{C}_i$ determines the optimal update step that minimizes the global cost function;
3) compensators in $\mathcal{C}_i$ actuate the system by updating their state $q_v, v \in \mathcal{C}_i$, while other compensators keep their state constant.



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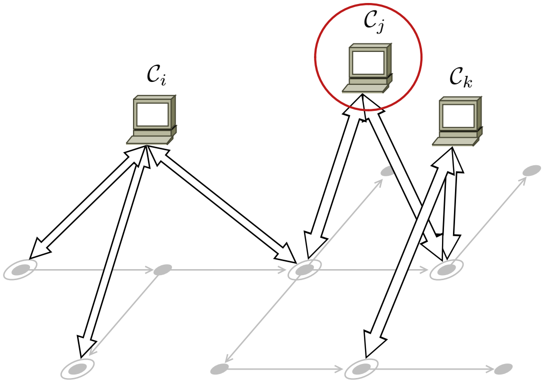

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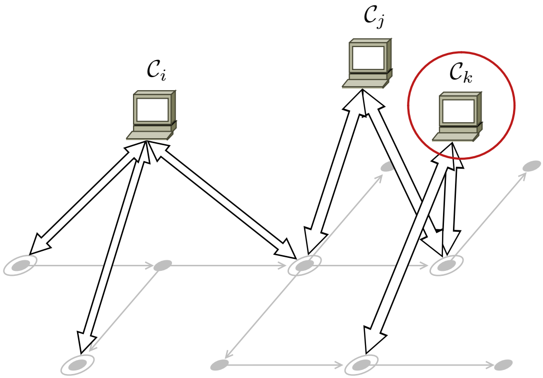


Computation of the optimal step for $\mathcal{C}_i$

The optimal update that has to be performed by cluster $\mathcal{C}_i$ is given by the (constrained) Newton step:

$$\begin{cases} q_h^{\text{opt},\ i} = q_h & \text{for each } h \notin \mathcal{C}_i, \\ q_h^{\text{opt},\ i} = q_h - \sum_{k \in \mathcal{C}_i} \Gamma_{hk}^{(i)} \nabla J_k & \text{for each } h \in \mathcal{C}_i, \end{cases}$$

where

- $\Gamma^{(i)}$ is function of the Hessian $\Re(X)$,
- ∇J is the gradient.

In general, these are global quantities.

However, according to the approximate model for the power system state, both $\Gamma_{hk}^{(i)}$ and ∇J_k can be obtained from local data.

Computation of the optimal step for $\mathcal{C}_i$

Hessian estimation

$\Gamma^{(i)}$ is a function of the electric distances (mutual effective impendances) between the microgenerators belonging to the cluster $\mathcal{C}_i$.

Gradient estimation

$\nabla J_k, k \in \mathcal{C}_i$, can be estimated from voltage measurements performed by the microgenerators that belong to $\mathcal{C}_i$.

To solve the subproblem faced by the supervisor of the cluster $\mathcal{C}_i$, only parameters and measurements from the microgenerators belonging to $\mathcal{C}_i$ are needed.

Resulting algorithm

We therefore obtained the following distributed control algorithm.

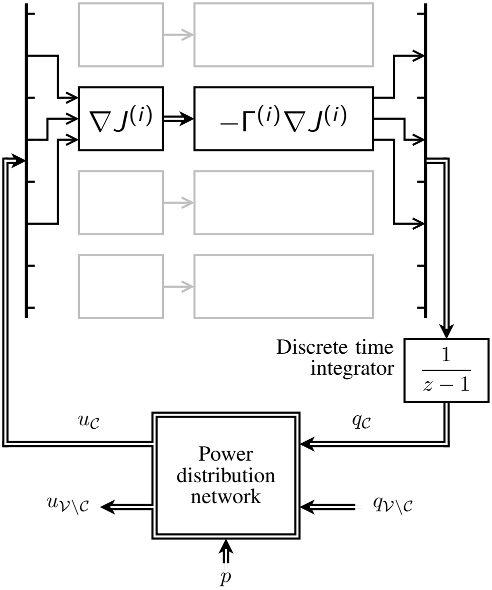
Offline initialization

Each supervisor computes $\Gamma^{(i)}$ according to the electric distance among compensators in the cluster.

Online iterative algorithm

1. a cluster $\mathcal{C}_i$ activates;
2. agents not in $\mathcal{C}_i$ hold their state constant;
3. agents in $\mathcal{C}_i$
   - 3.1 measure their voltage and estimate ∇J;
   - 3.2 compute the optimal update step $-\Gamma^{(i)} \nabla J$;
   - 3.3 update their state;

Resulting feedback law



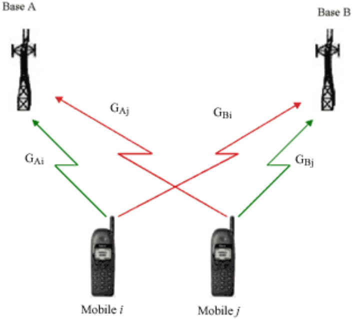
Remark

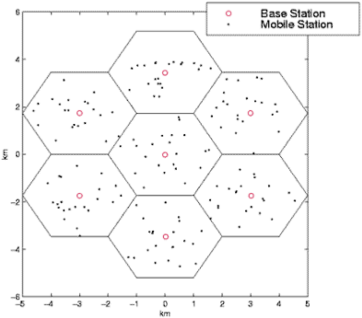
Feedback signals over the physical system

Other applications of distributed optimization share this same feature (radio power control, congestion avoidance protocols in data networks).

In these applications, the iterative tuning of the decision variables associated to each agent (radio power, transmission rate) depends on congestion indices that are function of the entire state of the system. However, these indices can be detected locally by each agent by measuring some feedback signals: error rates, delays, signal-to-noise ratios, etc.

Radio power control



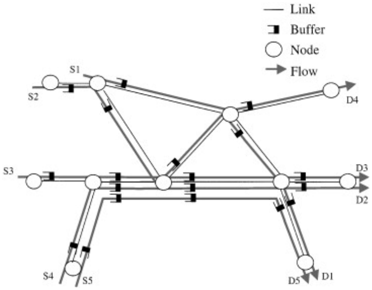


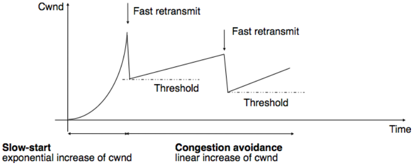
$$\text{SIR}_i = \frac{p_i}{\sum_{j \neq i} p_j w_{ij} + n_i G_{Ai}} \geq \text{SIR}^{\min}, \quad \forall i$$

Distributed radio power control algorithms consist in update laws for the transmitting power p_i in the form

$$p_i^+ = f_i \left(\text{SIR}_j, j \in \mathcal{N}_i \cup \{i\} \right).$$

Data network congestion avoidance protocol





$$\max_{x>0} \sum_r U_r(x_r) \quad \text{subject to } Ax \leq C$$

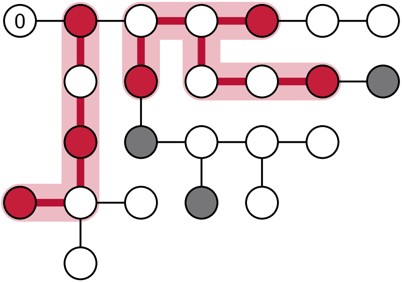
Congestion on a route r depends on the transmission rates of all routes which share a link with r, and which are generally unknown. Typical protocols adjust the rate x_r as a function of a feedback signal (e.g. delay, packet losses).

ORPF algorithm convergence

We characterized the convergence rate R as a function of

- grid topology and parameters
- clustering strategy.

The optimal strategy consists in choosing clusters which resembles the physical interconnection of the electric network.



Optimal clustering stategy

This result is interesting in the fact that it constrasts with the phenomena generally observed in gossip consensus algorithms, in which long-distance communications are beneficial for the rate of convergence.

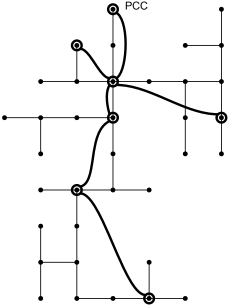
$$J = \frac{1}{|U_N|^2} q^T \,\Re(X)\, q \quad \text{subject to} \quad 1^T q = 0.$$

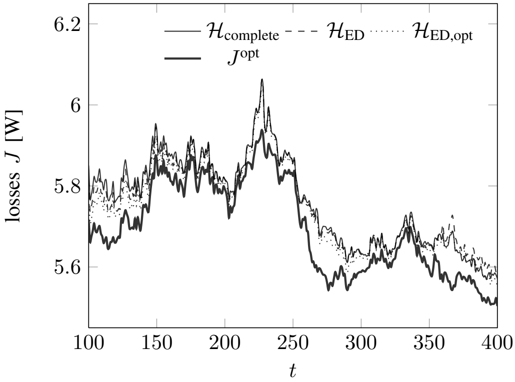
This is of course motivating, and suggests further investigation towards

- plug and play protocols,
- parallel implementation,
- communication over power lines.

Simulations

The algorithm behavior has been simulated on the IEEE 37 standard testbed.





CONCLUSIONS

Conclusions

Microgrid power flows model

The proposed approximate power flow model

- extends the DC model to generic line impedances
- allows to cast the problem into a well-known framework
- shows how to obtain system-wide information (gradient, hessian) from local measurements (voltages, electric distance).

Distributed gossip-like algorithm

The proposed strategy is based on

- asynchronous activation of the microgenerators
- interleaved sensing and actuation.

Its convergence is guaranteed, and its rate of convergence has been analyzed, yielding design rules to maximize performance.

Bolognani, S., and Zampieri, S. (2011).
Distributed control for optimal reactive power compensation in smart microgrids.
Extended version available online on http://automatica.dei.unipd.it
50th IEEE CDC, Orlando (FL), USA.

Bolognani, S., and Zampieri, S. (2012).
A distributed control strategy for reactive power compensation in smart microgrids.
To appear on IEEE Transactions of Automatic Control. Preprint available online on http://arxiv.org

Thanks!

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