Complex Energy Systems

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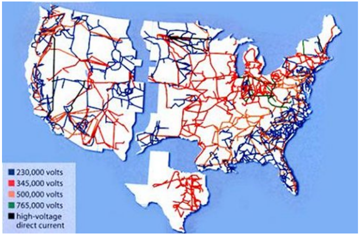
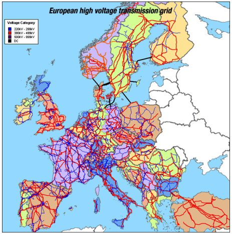
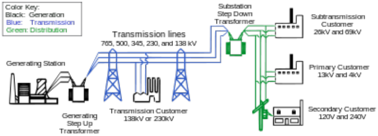
Lund, Oct 15, 2014

Power Grids



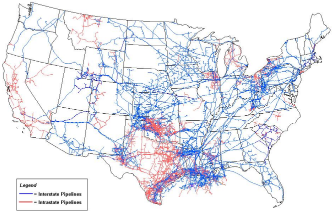




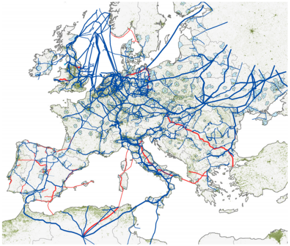
- Power Grids = the greatest engineering achievement of 20th century [IEEE]

- Require smart revolution in 21th century

Gas Grids



Source: Energy Information Administration, Office of Oil & Gas, Natural Gas Division, Gas Transportation Information System



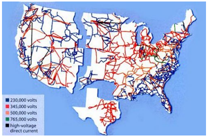
- “smart grids” are not limited to power, should also include other energy grids, e.g. gas grids
- gas networks are younger and less mature
- gas use is expected to grow

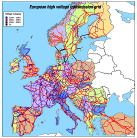
energy grids = electric+gas+heat

- Energy Hubs (local)
- Energy Interconnections (long distance)
- Vision of Future Energy Networks (green field)

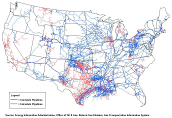
Theme(s) of the talk

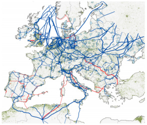
- Power and Gas Grids = Basic Energy Grids
- **Fluctuations** & perturbances test the grids
- Calling for new measures and understanding of **reliability**
- Need to deal with emerging **interdependencies**





power grids



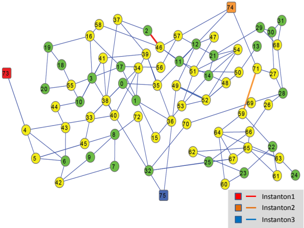


gas grids

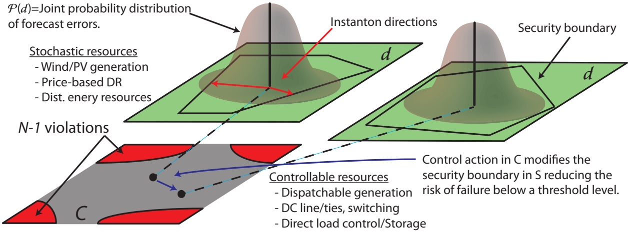
Outline

Reliability Measure of Power System Under Uncertainty

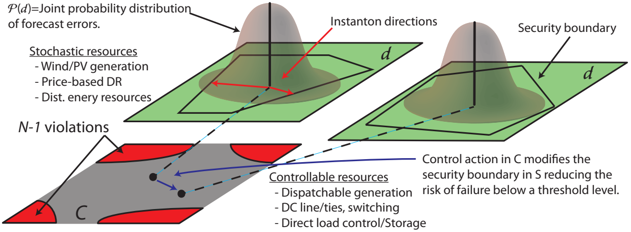
- Stochastic/uncontrollable participants (e.g. renewables) fluctuate
- Just the standard "N-1"-security gives no guarantees under uncertainty



Instantons in Power Systems: MC, F. Pan, M. Stepanov (2010); MC, FP, MS, R. Baldick (2011); S.S. Baghsorkhi, I. Hiskens (2012)



Reliability Measure of Power System Under Uncertainty



- **Step one (distance to failure)**: compute the instantons to find the total probability of stochastic failure = $\mathcal{P}_{\text{fail}}$
- **Step two**: if $\mathcal{P}_{\text{fail}} >$ threshold re-dispatch controllable resources so that $\mathcal{P}_{\text{fail}} <$ threshold at minimum cost

Step two — can be built in real-time operations

- e.g. E. Karangelos, P. Panciatici, L. Wehenkel, *Whither probabilistic security management for real-time operation of power systems?*, IREP 2013

Chance Constrained Re-dispatch

- Or ... instead of **Steps one and two** one can follow another path ⇒
- **Incorporate probabilistic security directly into optimization**

Chance Constrained Re-dispatch

- Or ... instead of **Steps one and two** one can follow another path ⇒
- **Incorporate probabilistic security directly into optimization**

D. Bienstock, MC, S. Harnett (Columbia/LANL) SIAM Review, Aug 2014
R. Bent, DB, MC http://arxiv.org/abs/1306.2972

- **CC-OPF** = make sure that generation is re-dispatched at minimum cost such that $\forall failures: \ \mathcal{P}_{\text{failure}} < \text{threshold}$

Related, independent work

- E. Sjodin, D. F. Gayme and U. Topcu, *Risk-Mitigated Optimal Power Flow for Wind Powered Grids*, ACC 2012.
- L. Roald, F. Oldewurtel, T. Krause and G. Andersson, *Analytical Reformulation of Security Constrained Optimal Power Flow with Probabilistic Constraints*, Proceedings of the Grenoble PowerTech, Grenoble, France, June 2013.
- M. Vrakopoulou, K. Margellos, J. Lygeros and G. Andersson, *A Probabilistic Framework for Reserve Scheduling and N-1 Security Assessment of Systems with High Wind Power Penetration*, to appear IEEE Transactions on Power Systems.

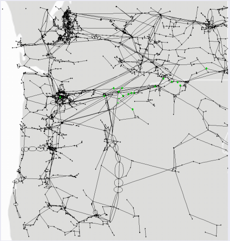
Standard re-dispatch = Optimum Power Flow

Constrained (thermal + generation limits) OPF

$$\min_{p,\theta} \underbrace{c(p)}_{\text{cost of generation}} \Bigg| \begin{array}{ll} B\theta = p - d & \text{[Power flows]} \\ \beta_{ij}(\theta_i - \theta_j) \le u_{ij},\ \forall (i,j) & \text{[Thermal limits]} \\ P_g^{min} \le p_g \le P_g^{max},\ \forall g & \text{[Generation constraints]} \end{array}$$

- p = vector of generations $\in \mathcal{R}^n$, d = vector of loads $\in \mathcal{R}^n$
- $B \in \mathcal{R}^{n\times n}$ – – bus susceptance matrix

- also called tertiary control; done by SO every 5-30 min
- DC-approximation [AC-generalizable]
- may also account for "standard" security (list of contingencies)

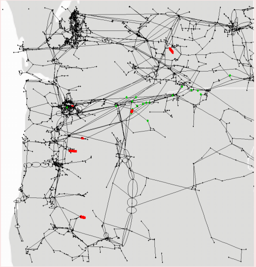


How does OPF handle (renewable) fluctuations?

- Automatic frequency control: primary [seconds] + secondary [AGC, 1-2 minute]
- Generator output varies up or down **proportionally** to **aggregate** change

Experiment: Bonneville Power Administration data, Northwest US

- data on wind fluctuations at planned farms
- with standard OPF, 7 lines exceed limit $\geq 8\%$ of the time



Want to improve the standard OPF

- standard automatic control [affine, possibly changing rates]
- aware of security (limits)
- not too conservative
- computationally practicable

OPF vs Chance Constrained-OPF

Standard OPF (Dispatch for the mean forecast, not aware of fluctuations)

$$\min_p \underbrace{c(p)}_{\text{cost of generation}} \quad \Big| \quad \begin{array}{c}\text{Power Flow Eqs.}\\ \text{Generation limits}\\ \text{Power Flow Thermal Limits}\end{array}$$

Chance Constrained OPF (fluctuations aware dispatch)

- $\min_{\bar{p},\alpha} \mathbb{E}\left[c(\bar{p},\alpha)\right]$ | Power Flow Eqs. [for mean forecast]; Chance Constraints for Generation; Chance Constraints for line Flows
- Chance Constraints for Line Flows:
  $\forall (i,j) \in \mathcal{E}: \quad \text{Prob}(|f_{ij}| > f_{ij}^{max}) < \varepsilon_{ij}$.
  Interpretation: overload is allowed for ϵ-fraction of "time".
- $\bar{p}$ - generation re-dispatch for beginning of the period; α - proportional rates for the period

Chance Constrained OPF (fluctuations aware dispatch)

- $\min_{\bar{p},\alpha} \mathbb{E}\left[c(\bar{p},\alpha)\right]$ | Power Flow Eqs. [for mean forecast]
  Generation satisfies Chance Constraints
  Line Power Flows satisfy Chance Constraints
- Chance Constraints for Line Flows:
  $\forall (i,j) \in \mathcal{E}: \quad \text{Prob}(|f_{ij}| > f_{ij}^{max}) < \varepsilon_{ij}$

CC-OPF detailed formulation

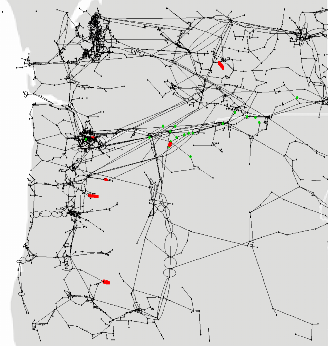
More Technical Details [it is NOT sampling/MCMC]

- Assuming **site-independent**, Gaussian fluctuations enables explicit evaluation [formula] of chance constraints for given $\bar{p}, \alpha$
- The resulting (after averaging) problem is a **convex (conic) optimization** [details]
- Constraint violations are few/sparse. **Cutting Plane** method greatly speeds up the optimization [details]

Back to motivating example

BPA case

- standard OPF: cost 235603, 7 lines unsafe $\geq$ 8% of the time
- CC-OPF: cost 237297, every line safe $\geq$ 98% of the time
- run time $=$ 9.5 seconds (one cutting plane!)



Back to motivating example

BPA case

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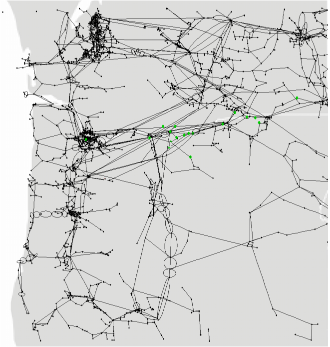


▸ Experiments with CC-OPF

- **CC-OPF succeeds where standard OPF fails**
- Cost of Reliability [CC-OPF saving over standard OPF]
- CC-OPF is not a naive fix. [Changes are nonlocal]
- ...

Performance of the Method

Performance of cutting-plane method on a number of large cases.

Case	Buses	Generators	Lines	Time (s)	Iterations	Barrier iterations
BPA	2209	176	2866	**5.51**	2	256
Polish1	2383	327	2896	**13.64**	13	535
Polish2	2746	388	3514	**30.16**	25	1431
Polish3	3120	349	3693	**25.45**	23	508

Rapid convergence on realistic networks

Typical convergence behavior of cutting-plane algorithm on a large instance.

Iteration	Max rel. error	Objective
1	1.2e-1	7.0933e6
4	1.3e-3	7.0934e6
7	1.9e-3	7.0934e6
10	1.0e-4	7.0964e6
12	8.9e-7	7.0965e6

details

Enhancements of CC-OPF

Out of Sample Tests – can handle either of the two cases

- True distribution is non-Gaussian, but our Gaussian distribution is close
- Parameters of the Gaussian distributions, μ_i, σ_i^2 are mis-estimated

details

Robust (ambigious) CC-OPF

- CC-OPF which is robust with respect to parameters of the Gaussian distribution from a range
- Allows convex tractable reformulation

"Non-linear" OPF & sync-CC OPF [R. Bent, D. Bienstock, MC 2013]

- Convex AC-OPF (losseless, constant voltage - based on an exercise from Boyd, Vandenberghe [book] details)
- Synchronization constrained CC-OPF (based on MC work with F. Dorfler & F. Bullo [PNAS, 2013] details)
- Voltage constraints — work in progress

Summary (CC-OPF) + Extensions:

- DC PF + affine control + independent fluctuations ⇒ conic (tractable) optimization
- Specialized cutting-plane algorithm proves effective
- Commercial solvers do not
- Algorithm efficient even in cases with thousands of buses/lines
- Algorithm can be made robust with respect to data errors
- Allows to account for synchronization constraints

Path Forward (work in progress)

- AC generalizations, convexifications (e.g. FDPF, Energy Function based appr.)
- Ramp Constraints
- Multi-stage CC-OPF
- combined CC-OPF & Unit Commitment

- Consider CC-OPF, or other type of dispatch, responding to uncertainty/wind
- Gas turbines [fast to ramp up/down, relatively clean] are

- **producers** of electricity: follow wind
- **consumers** of gas: inducing/transfering fluctuations/stress to the gas network

- Study interdependencies ...
- Start from analysis of gas system under uncertainty (e.g. caused by the wind-induced correlations)

2 min crash course on the hydro (gas) dynamics

- single pipe; not tilted (gravity is ignored); constant temperature
- ideal gas, $p \sim \rho$ – pressure and density are in a linear relation
- all fast transients are ignored – gas flow velocity is significantly slower than the speed of sound, $u \ll c_s$
- turbulence is modeled through turbulent friction; mass flow, $\phi = u\rho$, are averaged across the pipe's crossection

$$\left.\begin{array}{c}\underbrace{\partial_t \rho + \partial_x (u\rho) = 0}_{\text{conservation of mass}} \\ \underbrace{\partial_t(\rho u) + \partial_x(\rho u^2) + \partial_x p = -\frac{\rho u|u|}{2D} f}_{\text{conservation of energy}}\end{array}\right\} \approx\Rightarrow \begin{array}{c}\underbrace{c_s^{-2}\partial_t p + \partial_x \phi = 0}_{\text{conservation of mass}} \\ \underbrace{\partial_x p^2 + \frac{\beta}{D}\phi|\phi| = 0}_{\text{conservation of energy}}\end{array}$$

Approximations ... allowing to resolve flows analytically (lamp description)

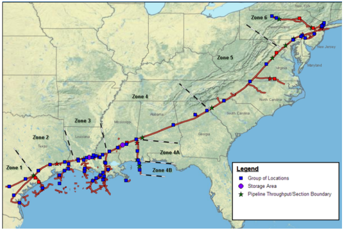
Stationary, balanced regime [standard]

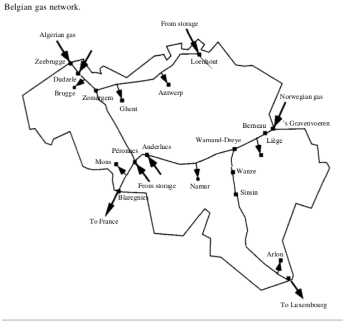
$$\phi = \text{const}, \quad p_{in}^2 - (p(x))^2 = x\beta\phi|\phi|/D$$

Unbalanced, linearized line-pack [non-standard]

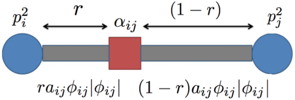
$$\phi = \phi_{\text{st}}(x) + \delta\phi(t,x), \quad p = p_{\text{st}}(x) + \delta p(t,x)$$

Gas Flows. Steady (balanced) Case.



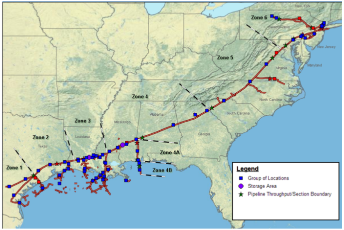


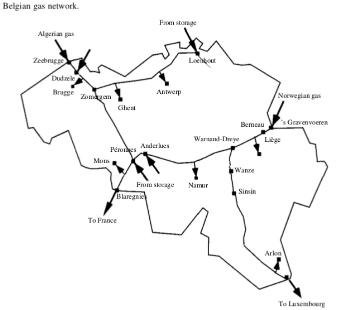
without compressors, $\alpha_{ij} = 1$



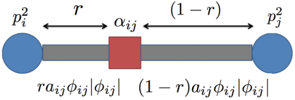
- Gas Flow Equations: $(\sum_i q_i = 0, \quad a_{ij} = L_{ij}\beta_{ij}/D_{ij})$
  $\forall (i,j): \quad p_i^2 - p_j^2 = a_{ij}\phi_{ij}^2$
  $\forall i: \quad q_i = \sum_{j:(i,j)\in\mathcal{E}} \phi_{ij} - \sum_{j:(j,i)\in\mathcal{E}} \phi_{ji}$

Gas Flows. Steady (balanced) Case.





with compressors, $\alpha_{ij} \lessgtr 1$



- Gas Flow Equations: $(\sum_i q_i = 0, \quad a_{ij} = L_{ij}\beta_{ij}/D_{ij})$

$$\forall (i,j): \quad \alpha_{ij}^2 = \frac{p_j^2 + (1-r)a_{ij}\phi_{ij}^2}{p_i^2 - ra_{ij}\phi_{ij}|\phi_{ij}|}$$

$$\forall i: \quad q_i = \sum_{j:(i,j)\in\mathcal{E}} \phi_{ij} - \sum_{j:(j,i)\in\mathcal{E}} \phi_{ji}$$

Optimal Gas Flow

Minimizing the cost of compression (∼ work applied externally to compress)

$$\min_{\alpha,p} \sum_{(i,j)} \frac{c_{ij}\phi_{ij}}{\eta_{ij}} \left(\alpha_{ij}^m - 1\right)^+ \Bigg| \quad \begin{array}{l} \forall (i,j): \quad \alpha_{ij}^2 = \frac{p_j^2 + (1-r) a_{ij}\phi_{ij}^2}{p_i^2 - r a_{ij}\phi_{ij}^2} \\ \forall i: \quad 0 \le \underline{p}_i \le p_i \le \overline{p}_i \\ \forall (i,j): \alpha_{ij} \le \overline{\alpha}_{ij} \end{array}$$

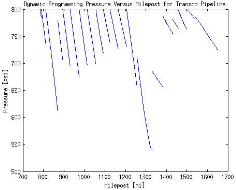
- $0 < m = (\gamma - 1)/\gamma < 1$, γ- gas heat capacity ratio (thermodynamics)

- The problem is **convex on trees** (many existing gas transmission systems are trees) $\Leftarrow$ through GeometricProgramming (log-function transformation)

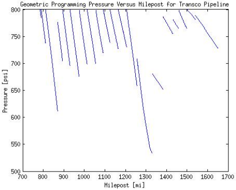
- **S. Misra, M. W. Fisher, S. Backhaus, R. Bent, MC, F. Pan**, *Optimal compression in natural gas networks: a geometric programming approach*, IEEE TCNS 2014

OGF experiments (Transco pipeline)

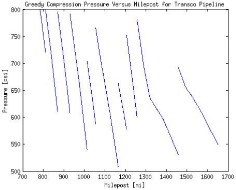
Dynamic Programming of (Wong, Larson '68)



Geometric Programming (ours)



Greedy Compression (current practice)

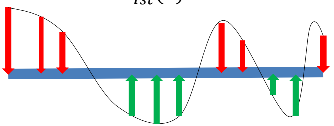


GP is advantageous over DP

- Exact = no-need to discretize.
- Faster. Allows distributed (ADMM) implementation.

- Convexity is lost in the loopy case. However, an efficient heuristics is available. [work in progress]
- This is only one of many possible OGF formulations. Another (Norvegian/European) example – maximize throughput.
- Major handicap of the formulation (ok for scheduling but) = did not account for the **line pack** (dynamics/storage in lines for hours) ⇒

Dynamic Gas Flows (with Line Pack) – Formulation



Steady (balanced) continuous profile
of gas injection/consumption

- $q(t,x) = q_{st}(x) + \xi(t,x), \quad \xi(t,x) \ll q_{st}(x)$
  - $q_{st}(x)$ is the forecasted consumption/injection of gas
  - $\xi(t,x)$ actual fluctuating/random profile of consumption/injection, e.g. **fluctuations due to gas power plants following wind turbines**

One dimensional (1+1) model – distributed injection/consumption and compression

- mass balance:
  $c_s^{-2}\partial_t p + \partial_x \phi = -q(t,x)$
- energy balance:
  $\partial_x p + \frac{\beta}{2d}\frac{\phi|\phi|}{p} = \gamma(x)p$
- $\gamma(x)$ – distributed compression – assumed known

- generalized to an arbitrary graph
- **S. Backhaus, MC, and V. Lebedev, PNAS submitted**

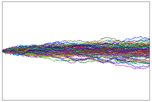
- Describe **spatio-temporal fluctuations of actual pressure** (unbalanced/line pack) on the top of the steady/optimized/inhomogeneous forecast

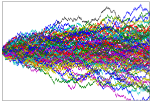
Dynamic Gas Flows (with Line Pack) – Solution

Analytic expressions for the line pack – assuming $\delta p(t) \ll p_{st}, \quad |\delta\phi| \ll |\phi_{st}|$

- analytical (!!) solution:
  - $\delta p(t,x) \approx -Z(x)\frac{c_s^2}{LY}\int_0^t dt' \int_0^L dx\, \xi(t',x)$
  - $Z(x) = \exp\left(\int_0^x dy\ \frac{\beta\phi_{st}(y)|\phi_{st}(y)|}{d\, p_{st}^2}\right), \quad Y = \int_0^L dx\ Z/L$

- $\delta p(t,x)$ is random zero mean Gaussian
- line pack jitters = grows "diffusively" with time
- the growth rate of the pressure fluctuations, $\sim Z(x)$, is non-uniform, depends (only) on the stationary solution
- $\mathbb{E}\left[\delta p(t,x)^2\right] \to \frac{c_s^4 \tau t}{L^2}\left(\frac{Z(x)}{Y}\right)^2 \iint_0^L dx_1\ dx_2 \mathbb{E}\left[\xi(t,x_1)\xi(t,x_2)\right]$





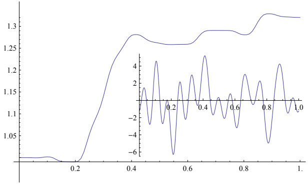
... all of the above is true ...

- When either correlation time or correlation scale of $\xi(t,x)$ is sufficiently short, i.e. $\tau \ll T$ (say minutes vs hours) or $l \ll L$ (say 10km vs 1000km)
- and $\xi(t,x)$ is zero mean and statistically stationary

Spatially Inhomogeneous **Line Pack Jitter**

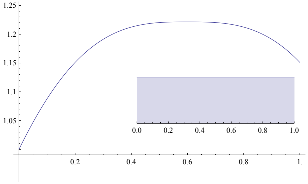
$$\mathbb{E}\left[\delta p(t,x)^2\right] \to \frac{c_s^4 \tau t}{L^2}\left(\frac{Z(x)}{Y}\right)^2 \iint_0^L dx_1\ dx_2 \mathbb{E}\left[\xi(t,x_1)\xi(t,x_2)\right]$$

Local maxima at the points of flow reversals

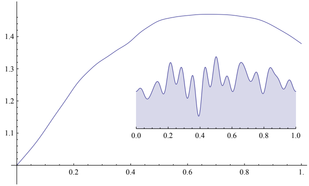


- $\sim Z^2(x)$ [controlling the "frozen" part of the pressure covariance] is shown
- $q_{st}(x)$ is shown in inset – distributed injection/consumption, $q_{st}(0) = q_{st}(L) = 0$
- $\gamma(x)$ is chosen to get $p_{st} = \text{const}$

Injection on two sides of the pipe
enhancement and shift of the maximum



uniform consumption



inhomogeneous consumption

Re-cap of the OGF & pressure reliability studies

- Geometric programming offers an efficient way of solving the steady/balanced OGF over tree structures
- Dynamic and stochastic GF (with line pack) is solved perturbatively. Shows diffusive, spatially inhomogeneous **line pack jitter** – extremal at the points of the flow reversal.

Path Forward

- Steady OGF over graphs with loops
- Other OGF formulations, e.g. max-throughput
- Generalize stochastic-line-pack-GF to discrete models with loops
- Extend it to reliability-aware stochastic and non-stationary optimizations

- Fluctuations in power sources (renewable and interdependencies) lead to more frequent (then in the past) interruptions
- In particular voltage faults
- Cleared faults do not cause major direct effect on balanced transmission

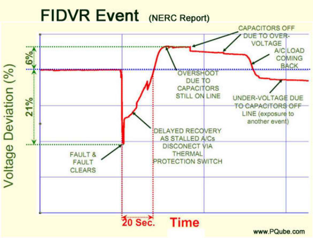
However, ...

- the faults (in spite of being cleared fast) may be of high risks for power distribution

Recordered Distribution/Transmission Voltage Events

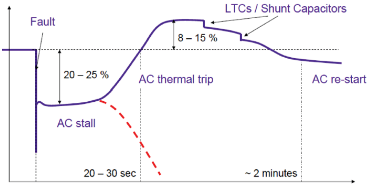
- TVA. Blistering Sat of Aug 22, 1987. Cascading Voltage Collapse in West Tennissee. Fault at 115KV switch. Cleared in 1s. Continued into 161KV and 500KV lines for 10-15s. Resulted in the loss of 700MW in Memphis. Motor loads stalled and drawn large amount of reactive power even after the fault was cleared.
- 1988 event in Florida reported in "Air Conditioner Respond to Transmission Fault" by J. W. Shaffer in 1997 ... "In the last ten years there have been at least eight events in which normally cleared (in 2-3 cycles) multi-phase events in Southern Florida have caused a significant drop in customer load (200-825MW)."
- 1990 Egypt ... 1999 metro area Atlanta, Arizona, Southern California ... NERC Planning Committee White Paper on "Fault Induced Delayed Voltage Recovery" by Transmission Issues Subcommittee

- delays (between cause and the result)
- nonlinearity of loads plays a significant role
- many inductive motors simultaneously affected
- initiated (fault) at the transmission-to-distribution interface, maturates within distribution, cascades into transmission



Typical FIDVR Following a 230-kV Transmission Fault in Southern California

Modeling extended FIDVR C.Duclut, S.Backhaus & MC (PRE '12)



courtesy of D. Kostyrev and B. Lesieutre

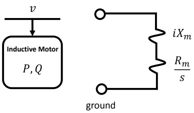
- Observed in feeders with many induction motors (air-conditioning)
- Uncontrolled depressed voltage can spread causing a larger outage
- Hypothesis (Hiskens, Lesieutre, Chassin, $\cdots$): the events are caused by many air conditioners stalled
- Modeling the event is a challenge

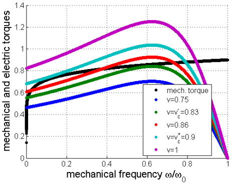
Our Contribution - Modeling of FIDVR over extended feeder

- Observation (simulations – consistent with measurements): soliton-like propagation of "stalled" phase/front
- Coarse-grained (reduced) PDE modeling of the "extended" FIDVR

Modeling Individual Motor   Popovic, Hiskens, Hill '98

minimal model of the motor





$$P = \frac{sR_m v^2}{R_m^2 + s^2 X_m^2}$$

$$Q = \frac{s^2 X_m v^2}{R_m^2 + s^2 X_m^2}$$

$$M\frac{d}{dt}\omega = \frac{P}{\omega_0} - T_0(\omega/\omega_0)^\alpha \text{ (dynamics)}$$

$$s = 1 - \omega/\omega_0$$

s is the slip against the base frequency)

v is the voltage at the motor

P, Q are real and reactive power consumed by the motor

T_0, α torque constant and scaling coefficient

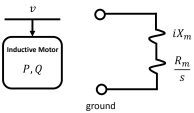
R_m, X_m resistance and inductance of the motor

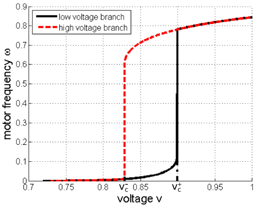
Explanation for "lumped" FIDVR

- Hysteresis: The motor is trapped in the stalled (low-voltage) state!
- First order phase transition. Bifurcation (stability). Spinodal points.

Modeling Individual Motor — Popovic, Hiskens, Hill '98

minimal model of the motor





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Explanation for "lumped" FIDVR

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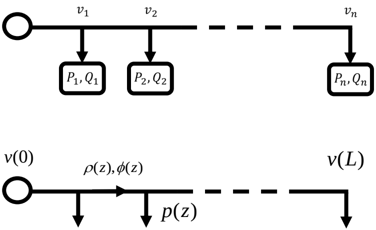
Feeder with Many (distributed) Inductive Motors



$$\partial_z \rho = -p - r\frac{\rho^2 + \phi^2}{v^2}$$

$$\partial_z \phi = -q - x\frac{\rho^2 + \phi^2}{v^2}$$

$$v\partial_z v = -(r\rho + x\phi)$$

$$p = \frac{s r_m v^2}{r_m^2 + s^2 x_m^2}$$

$$q = \frac{s^2 x_m v^2}{r_m^2 + s^2 x_m^2}$$

$$\mu \frac{d}{dt}\omega = \frac{p}{\omega_0} - \tau_0 \left(\frac{\omega}{\omega_0}\right)^\alpha$$

$$v(0) = 1, \ \rho(L) = \phi(L) = 0$$

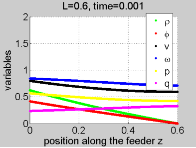
- Spatially-continuous version of Dist.Flow [Baran, Wu (1989)]

Reduced model of the "extended" feeder

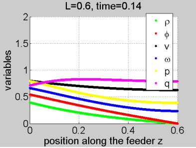
Easy to analyze dynamics: PDE.

Dynamics/Transitions in an Extended Feeder (I)

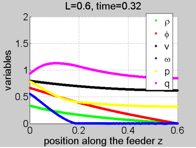
Example of a Large Fault → feeder is stalled (Movie Large Fault)



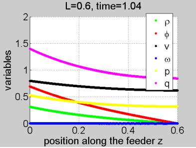
(a)



(b)



(c)

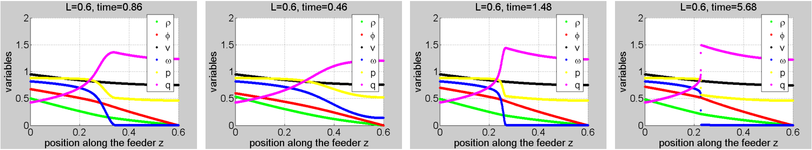


(d)

- (a) Pre fault
- (b) Immediately past fault
- (c) Later in the process
- (d) The feeder is fully stalled

Dynamics/Transitions in an Extended Feeder (II)

Example of a Small Fault → feeder is partially stalled (Movie Small Fault)



(a) (b) (c) (d)

- (a) Immediately past fault
- (b) Later in the process
- (c) Front advances
- (d) Stabilized, part. stalled

dynamics of restoration

- The 1+1 (space+time) continuous model of distribution
- Integrating multiple bi-stable individual motors into power flow
- Emergence of multiple spatially-extended states/transitions

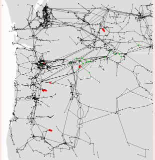
Conclusions Drawn from Experiments/Numerics concern

- **Hysteresis**
- **Self-Similar Transients**

... to be done ...

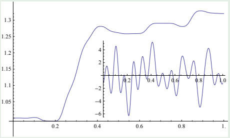
- Inhomogeneity (disorder), stochasticity (noise): what is the probability that the feeder with a given level of disorder will recover?
- Effects of other devices, e.g. distributed generation and control (PV) ...
- Possible cascade – from feeder to feeder (within substation) ... to transmission
- What is the least control effort needed to avoid a FIDVR event/cascade following a given type of fault? $\Leftarrow$ Voltage control through
  - tap changers, e.g. incorporating in the framework of "primary voltage control of active distributed networks" by Christakou, Tomozei, Le Boudec, Paolone (2014)
  - inverters [distributed power electronics]

Chance Constrained OPF



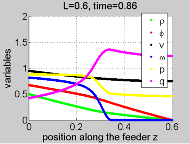
- New reliability measure for uncertainty – the instantons
- Efficient and Scalable Chance-Constrained OPF

Gas Reliability with Line Pack



- Geometric/Signomial Programming for Optimum Gas Flow
- Line-Pack Jitter of Pressure Fluctuations

Distributed Fault Induced Delayed Voltage Recovery



- Soliton-like front describing FIDVR
- PDE approach $\Rightarrow$ coarse-grained estimation

Take Home High-level message illustrated today on a few examples

New Science of Complex Power/Energy System Engineering

- Old approach focused on individual devices, deterministic – remained valid ... but
- New complexities (renewables, fluctuations, interdependencies) need to be controlled through ...
- Better understanding of the multi-scale, probabilistic science (applied physics/math, operation research, other IT disciplines) ...
- To enable better practical control and optimization of tomorrow grids

Take Home High-level message illustrated today on a few examples

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- To enable better practical control and optimization of tomorrow grids

Thank You!

DC-approximation

(0) The amplitude of the complex potentials are all fixed to the same number (unity, after trivial re-scaling): $\forall a: \quad u_a = 1$.

(1) $\forall \{a, b\}: \quad |\varphi_a - \varphi_b| \ll 1$ - phase variation between any two neighbors on the graph is small

(2) $\forall \{a, b\}: \quad r_{ab} \ll x_{ab}$ - resistive (real) part of the impedance is much smaller than its reactive (imaginary) part. Typical values for the r/x is in the $1/27 \div 1/2$ range.

(3) $\forall a: \quad p_a \gg q_a$ - the consumed and generated powers are mainly real, i.e. reactive components of the power are much smaller than their real counterparts

It leads to

- Linear relation between powers and phases (at the nodes): $\hat{B}\varphi = \boldsymbol{p}$
- Losses are ignored: $\sum_a p_a = 0$
- B - graph Laplacian constructed of line susceptances

◂ DC-OPF

Frequency Control (quasi-static proxy)

For each generator i, two parameters:

- $\overline{p_i}$ = mean output
- α_i = response parameter

Real-time output of generator i:

$$p_i = \overline{p}_i - \alpha_i \sum_j \Delta\omega_j$$

where $\Delta\omega_j$ = change in output of renewable j (from mean).

$$\sum_i \alpha_i = 1$$

$\sim$ primary + secondary control

Chance Constrained OPF

Computing line flows

wind power at bus i: $\mu_i + \boldsymbol{w}_i$

DC approximation

- $B\boldsymbol{\theta} = \overline{p} - d$
  $+(\mu + \boldsymbol{w} - \alpha \sum_{i \in G} \boldsymbol{w}_i)$
- $\boldsymbol{\theta} = B^+(\bar{p} - d + \mu) + B^+(I - \alpha e^T)\boldsymbol{w}$
- flow is a linear combination of bus power injections:

$$\boldsymbol{f_{ij}} = \beta_{ij}(\boldsymbol{\theta}_i - \boldsymbol{\theta}_j)$$

◂ Chance Constrained OPF

Computing line flows

$$\boldsymbol{f_{ij}} = \beta_{ij} \left((B_i^+ - B_j^+)^T (\bar{p} - d + \mu) + (A_i - A_j)^T \boldsymbol{w} \right),$$

$$A = B^+ (I - \alpha e^T)$$

Given distribution of wind can calculate moments of line flows:

- $E \boldsymbol{f_{ij}} = \beta_{ij} (B_i^+ - B_j^+)^T (\bar{p} - d + \mu)$
- $var(\boldsymbol{f_{ij}}) := s_{ij}^2 \geq \beta_{ij}^2 \sum_k (A_{ik} - A_{jk})^2 \sigma_k^2$
  (assuming independence)
- and higher moments if necessary

Chance Constrained OPF

Chance constraints to deterministic constraints

- chance constraint: $P(\boldsymbol{f_{ij}} > f_{ij}^{max}) < \epsilon_{ij}$ **and** $P(\boldsymbol{f_{ij}} < -f_{ij}^{max}) < \epsilon_{ij}$
- from moments of $\boldsymbol{f_{ij}}$, can get conservative approximations using e.g. Chebyshev's inequality
- for Gaussian wind, can do better, since $\boldsymbol{f_{ij}}$ is Gaussian :

$$|E\boldsymbol{f_{ij}}| + var(\boldsymbol{f_{ij}})\phi^{-1}(1 - \epsilon_{ij}) \leq f_{ij}^{max}$$

◂ Chance Constrained OPF

Formulation [convex!]:

Choose mean generator outputs and control to minimize expected cost, with the probability of line overloads kept small.

$$\min_{\overline{p},\alpha} \mathbb{E}[c(\overline{p})]$$

$$\text{s.t.} \sum_{i \in G} \alpha_i = 1, \;\; \alpha \geq 0$$

$$B\delta = \alpha, \delta_n = 0$$

$$\sum_{i \in G} \overline{p}_i + \sum_{i \in W} \mu_{\mathbf{i}} = \sum_{i \in D} d_i$$

$$\overline{f}_{ij} = \beta_{ij}(\overline{\theta}_i - \overline{\theta}_j),$$

$$B\overline{\theta} = \overline{p} + \mu - d, \;\; \overline{\theta}_n = 0$$

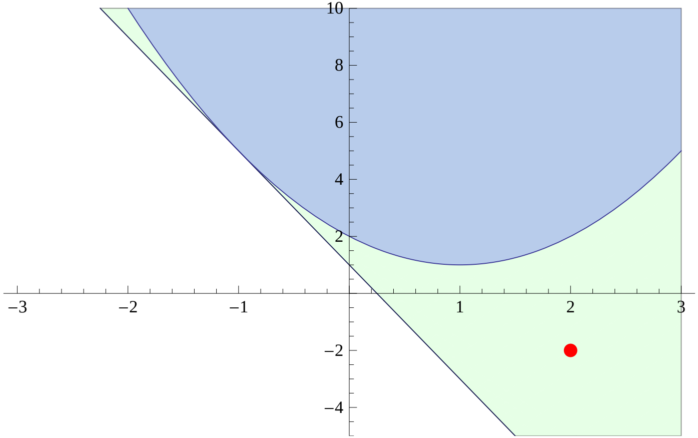
$$s_{ij}^2 \geq \beta_{ij}^2 \sum_{k \in W} \sigma_{\mathbf{k}}^{\mathbf{2}} (B_{ik}^+ - B_{jk}^+ - \delta_i + \delta_j)^2$$

$$|\overline{f}_{ij}| \; + \; s_{ij}\phi^{-1}\left(1 - \epsilon_{ij}\right) \leq f_{ij}^{max}$$

◂ Chance Constrained OPF

Cutting-Plane Method

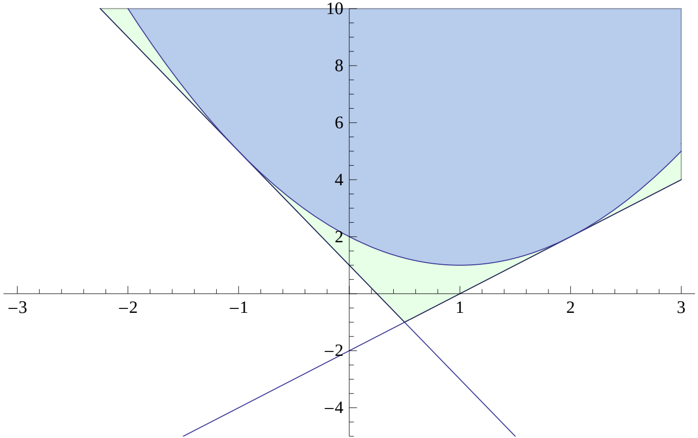
New Solutions still violates conic constraint



Chance Constrained OPF

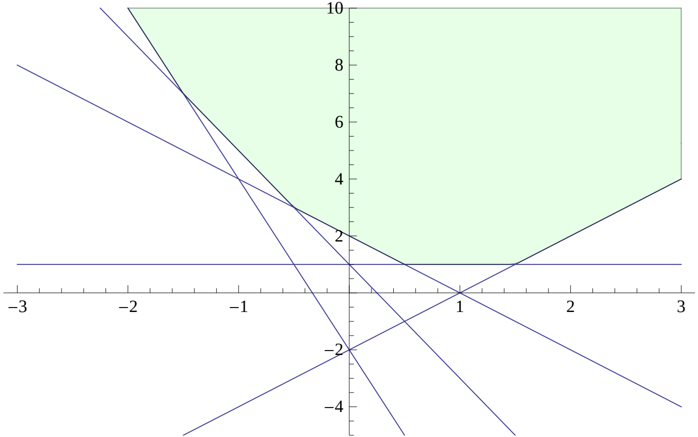
Cutting-Plane Method

Separate again



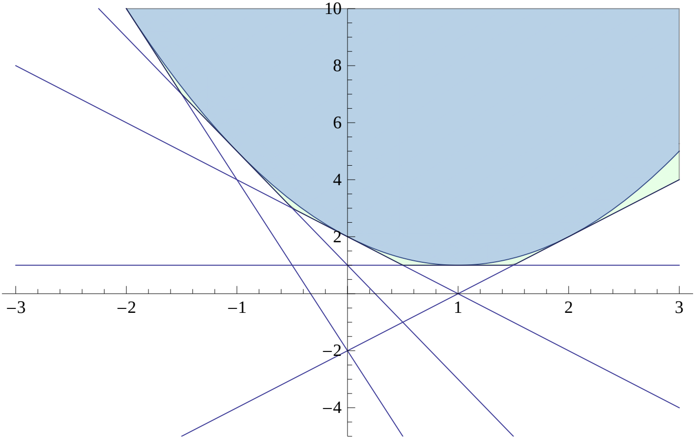
Cutting-Plane Method

We might end up with many linear constraints



Cutting-Plane Method

... which approximate the conic constraint



◂ Chance Constrained OPF

Polish 2003-2004 winter peak case

- 2746 buses, 3514 branches, 8 wind sources
- 5% penetration and $\sigma = .3\mu$ each source

CPLEX: the optimization problem has

- 36625 variables
- 38507 constraints, 6242 conic constraints
- 128538 nonzeros, 87 dense columns

◂ CC-OPF Performance

CPLEX:

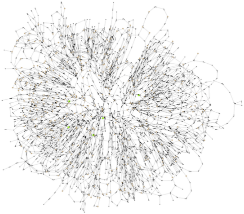
- total time on 16 threads = 3393 seconds
- "optimization status 6"
- solution is wildly infeasible

Gurobi:

- time: 31.1 seconds
- "Numerical trouble encountered"

◂ CC-OPF Performance

Polish 2003-2004 case
CPLEX: “opt status 6”
Gurobi: “numerical trouble”



Example run of cutting-plane algorithm:

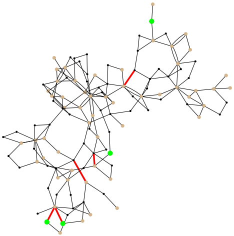
Iteration	Max rel. error	Objective
1	1.2e-1	7.0933e6
4	1.3e-3	7.0934e6
7	1.9e-3	7.0934e6
10	1.0e-4	7.0964e6
12	8.9e-7	7.0965e6

Total running time: 32.9 seconds

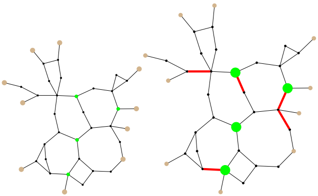
◂ CC-OPF Performance

Experiments with CC-OPF (I)

- **CC-OPF succeeds where standard OPF fails**
- 118-bus case with four wind farms. Standard OPF–Lines in red exceed their limit 8% or more of the time. CC-OPF–finds solution with significantly smaller risk of overload.



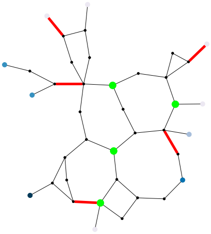
- **Cost of Reliability** [CC-OPF saving over standard OPF]
- 39-bus case under standard solution. Even with a 10% buffer on the line flow limits (for the average wind), five lines exceed their limit over 5% of the time with 30% penetration (right). The penetration must be decreased to 5% before the lines are relieved, but at great cost (left). The CC-OPF model is feasible for 30% penetration at a cost of 264,000. The standard solution at 5% penetration costs 1,275,020 almost 5 times as much.



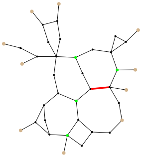
◂ BPA Experiment

Experiments with CC-OPF (II)

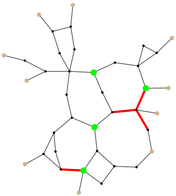
- **CC-OPF is not a naive fix.** (Changes are nonlocal.)
- 39-bus case. Darker shades of blue indicating generators with greater change from CC-OPF to standard OPF.



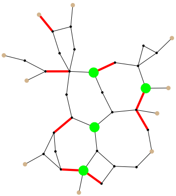
- **What is the penetration that can be tolerated** (without upgrading)?
- 39-bus case. Three levels of penetration. Standard OPF is infeasible for three level of penetrations. CC-OPF is infeasible only with the penetration level $> 30+\%$.



.1% wind



8% wind

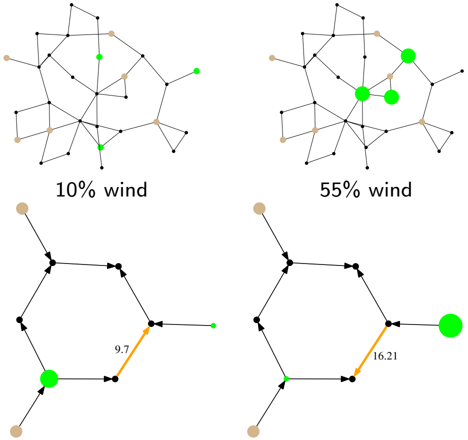


30% wind

◂ BPA Experiment

Experiments with CC-OPF (III)

- **Which sites to place wind-farms?**
- 30 bus case with three wind farms. Placement on the right is preferable.
- CC-OPF finds the nodes where the entire network is less susceptible to fluctuations.



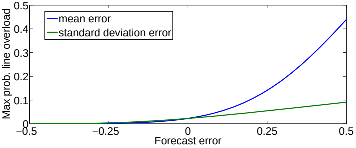
- CC-OPF valid configurations may **show significant (allowed!) variability**, e.g. flow reversal.
- 9-bus case, 25% average penetration - two significantly different flows.

BPA Experiment

Out of Sample Tests

Distribution	Max. prob. violation
Normal	0.0227
Laplace	0.0297
logistic	0.0132
Weibull, $k = 1.2$	0.0457
Weibull, $k = 2$	0.0355
Weibull, $k = 4$	0.0216
t location-scale, $\nu = 2.5$	0.0165
Cauchy	0.0276

Maximum probability of overload for out-of-sample tests. These are a result of Monte Carlo testing with 10,000 samples on the BPA case, solved under the Gaussian assumption and desired maximum chance of overload at 2.27%.



BPA case solved with average penetration at 8% and standard deviations set to 30% of mean. The maximum probability of line overload desired is 2.27%, which is achieved with 0 forecast error on the graph. Actual wind power means are then scaled according to the x-axis and maximum probability of line overload is recalculated (blue). The same is then done for standard deviations (green).

Enhancements of CC-OPF

Constant voltage, lossless, security constrained OPF:

$$\min_{p,\theta} \; c(p)$$

s.t.

$$\sum_{j:ij\in\mathcal{L}} \beta_{ij}\sin(\theta_i-\theta_j) \;=\; p_i \;-\; d_i \qquad \forall i\in\mathcal{B}$$

$$|\beta_{ij}\sin(\theta_i-\theta_j)| \;\le\; u_{ij} \quad \text{for each line } ij$$

$$P_g^{min} \;\le\; p_g \;\le\; P_g^{max} \quad \text{for each generator } g$$

Can one convexify this formulation of OPF?

CC-OPF enhancements

PF through Optimization [lossless, constant voltage]

Boyd & Vandenberghe (add. ex. for convex opt. – 2012):

Suppose you solve the **convex optimization problem**:

$$\min_{\rho\text{ - line flows}} \underbrace{\sum_{ij\in\mathcal{L}} \beta_{ij}\Psi(\rho_{ij})}_{\text{reactive losses in lines}} \quad , \quad \Psi(\rho) = \int_{-1}^{\rho} \arcsin(y)\, dy$$

$$\text{s.t.} \quad \underbrace{\sum_{j:ij\in\mathcal{L}} \beta_{ij}\rho_{ij} - \sum_{j:ji\in\mathcal{L}} \beta_{ij}\rho_{ji} = p_i - d_i}_{\text{network flow conservation}} \qquad \forall i \in \mathcal{B} \quad (*)$$

$$|\rho_{ij}| < 1 \quad \text{for each line } ij$$

Then: If θ_i is the optimal dual for (*), $\rho_{ij} = \sin(\theta_i - \theta_j)$.

The opt. is dual to the Energy Function opt.

How can we incorporate this methodology into OPF-type problems?

CC-OPF enhancements

Theorem: "Exact" AC-OPF [BBC 2013]

Suppose you solve the **convex optimization problem**:

$$\min_{p,\rho,\delta \geq 0} \quad c(p) \; + \; D \sum_{ij \in \mathcal{L}} \beta_{ij} \Psi(\rho_{ij}) \; - \; K \sum_{ij \in \mathcal{L}} \beta_{ij} \log(\delta_{ij})$$

s.t.

$$\sum_{j:ij \in \mathcal{L}} \beta_{ij} \rho_{ij} \; - \; \sum_{j:ji \in \mathcal{L}} \beta_{ij} \rho_{ji} \; = \; p_i \; - \; d_i \qquad \forall i \in \mathcal{B} \quad (**)$$

$$|\rho_{ij}| \; + \; \min\{1, u_{ij}/\beta_{ij}\}\delta_{ij} < \; \min\{1, u_{ij}/\beta_{ij}\} \quad \text{for each line } ij$$

$$P_g^{min} \; \leq \; p_g \; \leq \; P_g^{max} \quad \text{for each generator } g$$

For **appropriate** positive constants D (small) and K (large). Then if a feasible solution is found

- The optimal ρ_{ij} are approximate optimal flows [with line flow limits obeyed]
- $\rho_{ij} \approx \sin(\theta_i - \theta_j)$ $\quad \theta =$ optimal duals to (**)

▸ CC-OPF enhancements

AC-OPF [loseless, constant voltage] formulation

$$\min_{p,\vartheta} \; c(p)$$

s.t.

$$\sum_{j:ij\in\mathcal{L}} \beta_{ij}\sin(\theta_i - \theta_j) \;=\; p_i \;-\; d_i \qquad \forall i \in \mathcal{B}$$

$$|\sin(\theta_i - \theta_j)| < u_{ij}/\beta_{ij} \quad \text{for each line } ij$$

▸ CC-OPF enhancements

Based on Dörfler, Chertkov, Bullo 2013: an approximation

$$\min_{p,\vartheta} \quad c(p)$$

s.t.

$$\sum_{j:ij\in\mathcal{L}} \beta_{ij}(\vartheta_i - \vartheta_j) \;=\; p_i \;-\; d_i \qquad \forall i \in \mathcal{B}$$

$$|\vartheta_i - \vartheta_j| < \; \min\{1, u_{ij}/\beta_{ij}\} \quad \text{for each line } ij$$

[criterium for existence of solution, assumes strong damping]

Sync in Pics

- The ϑ are auxiliary variables only.
- Exact on trees, very accurate for almost all realistic cases tested
- In experiments, $\beta_{ij}(\vartheta_i - \vartheta_j)$ provides a close approximation to the lossless (active) AC power flow on each line ij
- (But does not provide phase angles)

Incorporation into chance-constrained problem:

- On any line ij, we replace $\sin(\theta_i - \theta_j)$ with the quantity $\vartheta_i - \vartheta_j$
- So 'sync' constraint $|\sin(\theta_i - \theta_j)| \le \gamma_{ij}$ becomes $|\vartheta_i - \vartheta_j| \le \gamma_{ij}$
- But in **either** case the constraint is stochastic

Results in a (conic) convex optimization

- **Chance-constrained version:** $P(|\vartheta_i - \vartheta_j| > \gamma_{ij}) < \epsilon_{ij}$

All (thermal, gen., sync) Chance Constraints accounted

- Results in the convex (conic) optimization
- Similar to DC CC-OPF – extra sync Chance-constraints added

CC-OPF enhancements

Chance-constrained, thermal and sync-aware (approximate) OPF:
Choose mean generator outputs and control to minimize expected cost, with the probability of line overloads and phase angle excursions kept small. (abridged)

$$\min_{\overline{p},\alpha} \mathbb{E}[c(\overline{p})]$$

$$\text{s.t.} \sum_{i\in G} \alpha_i = 1, \ \alpha \geq 0$$

$$B\delta = \alpha$$

$$\sum_{ij\in\mathcal{L}} \beta_{ij}(\bar{\vartheta}_i - \bar{\vartheta}_j) = \overline{p_i} + \mu_{\mathbf{i}} - d_i$$

$$P(\beta_{ij}|\boldsymbol{\vartheta_i} - \boldsymbol{\vartheta_j}| > u_{ij}) \leq \epsilon_1 \quad \text{for each line } ij$$

$$P(|\boldsymbol{\vartheta_i} - \boldsymbol{\vartheta_j}| > \gamma_{ij}) \leq \epsilon_2 \quad \text{for each line } ij$$

$$P(\boldsymbol{p_g} < P_g^{min} \text{ or } P_g^{max} < \boldsymbol{p_g}) \leq \epsilon_3 \quad \text{for each generator } g$$

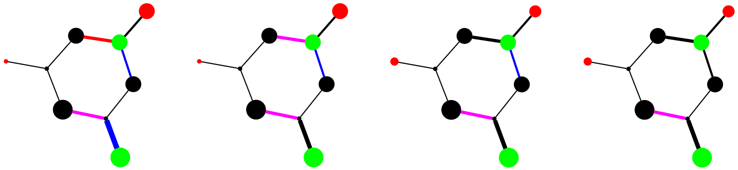
$$\boldsymbol{\epsilon_2 \ll \epsilon_3 \ll \epsilon_1}$$

Again: a conic optimization problem

CC-OPF enhancements

Thermal and Sync Aware CC-OPF: Experiments (I)

Competition of sync and thermal risks guides iterations



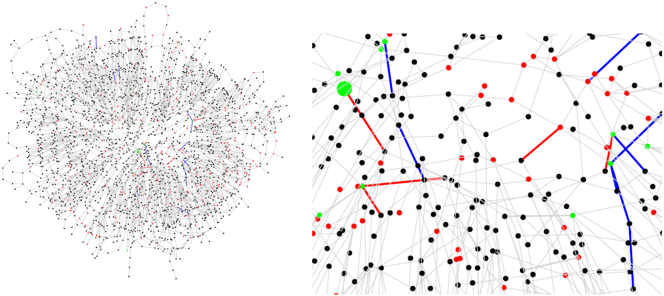
- The case of $\bar{p}_{ij}/\beta_{ij} \leq 1$ but $\varepsilon = 10^{-4} \ll \epsilon_{ij} = 10^{-2}$.
- 1st, 8th, 11th and 13th (final) iteration steps shown.
- Nodes: Loads = black, wind farms = green, regular generators = red
- Lines: sync+therm = red, only sync = magenta, only therm = blue, no viol. = black
- Scaling – with actual values or means

CC-OPF enhancements

Thermal and Sync Aware CC-OPF: Experiments (II)

Pattern(s) of Sync Warnings

- Qualitative value in studying the warning patterns

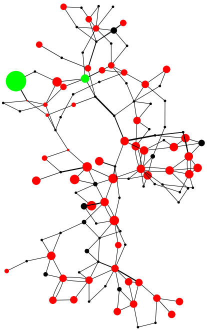


Polish case completed in 11 iterations. Sync overload dominated. Red lines = sync. overloaded with probability $\in [10^{-4}; 10^{-2}]$. Blue lines = weaker overload. Scaling according to cons/prod and mean flows within the optimal solution.

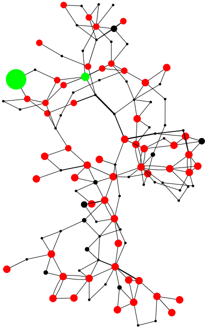
CC-OPF enhancements

Thermal and Sync Aware CC-OPF: Experiments (III)

Sensitivity of the optimal solution to risk awareness



9 cutting plane iterations, both sync and thermal conditions violated [less uniform]

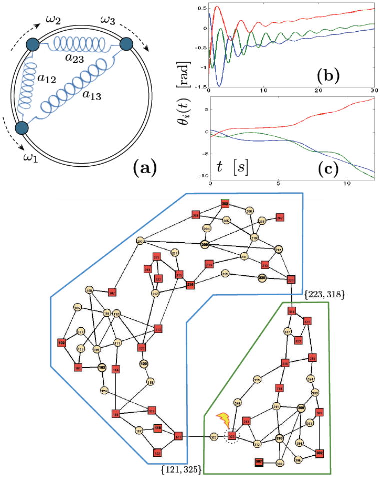


21 cutting plane iterations, only sync conditions violated

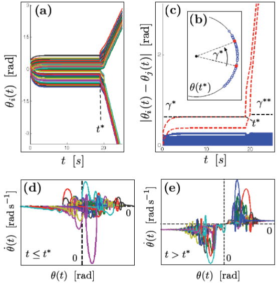
- two slightly different config. of loads
- results distinctly different [cost and distr. of gen.]
- red – regulated generation
- green – renewables [mean]

CC-OPF enhancements

Sync in Pics



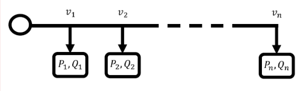
- from F. Dörfler, M. Chertkov, and F. Bullo, PNAS 2013



◂ Sync-constrained OPF

Dist(ributed) Flow Representation [Baran, Wu '89]

graph-linear Element $k = 1, \cdots, N$ of the distribution feeder

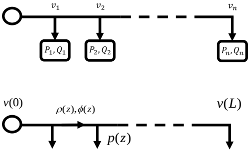


$$k = 0, \cdots, N, \quad v_0 = 1$$
$$P_{n+1} = Q_{n+1} = 0$$

$$P_{k+1} - P_k = p_k - r_k \frac{P_k^2 + Q_k^2}{v_k^2}$$
$$Q_{k+1} - Q_k = q_k - x_k \frac{P_k^2 + Q_k^2}{v_k^2}$$
$$v_{k+1}^2 - v_k^2 =$$
$$-2(r_k P_k + x_k Q_k) - (r_k^2 + x_k^2)\frac{P_k^2 + Q_k^2}{v_k^2}$$

- nonlinear AC over a line
- generalizable to a tree
- P_k, Q_k real and reactive powers flowing through the segment k
- p_k, q_k, v_k powers injected/consumed and voltage at the bus k

Continuum (one dimensional) static power flows



ODE with mixed boundary conditions
$v(0) = 1,\ \rho(L) = \phi(L) = 0$

From Algebraic Eqs. on a (linear) Graph to Power Flow ODEs

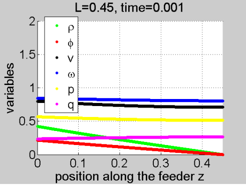
$$0 = \underbrace{p + \beta \partial_r \left(v^2 \partial_r \theta\right) + gv \left(\partial_r^2 v - v \left(\partial_r \theta\right)^2\right)}_{\text{balance of real power}}, \qquad 0 = \underbrace{q + \beta v \left(\partial_r^2 v - v \left(\partial_r \theta\right)^2\right) - g \partial_r \left(v^2 \partial_r \theta\right)}_{\text{balance of reactive power}}$$

$$\rho = \underbrace{-\beta v^2 \partial_r \theta - gv \partial_r v}_{\text{real power density flowing through the segment}}, \qquad \phi = \underbrace{-\beta v \partial_r v + gv^2 \partial_r \theta}_{\text{reactive power density flowing though the segment}}$$

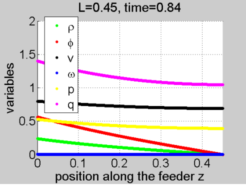
$$0 = \underbrace{p}_{\text{real consumption}} - \overbrace{\partial_r \rho}^{\text{real transport}} - \underbrace{r \frac{\rho^2 + \phi^2}{v^2}}_{\text{real dissipation}}, \quad 0 = \underbrace{q}_{\text{reactive consumption}} - \overbrace{\partial_r \phi}^{\text{reactive transport}} - \underbrace{x \frac{\rho^2 + \phi^2}{v^2}}_{\text{reactive dissipation}}$$

Dynamics/Transitions in Distributed Feeder (III)

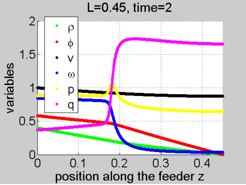
Example of a Short Fault (↓, ↑ to full recovery) (Movie Recovery)



(a)



(b)



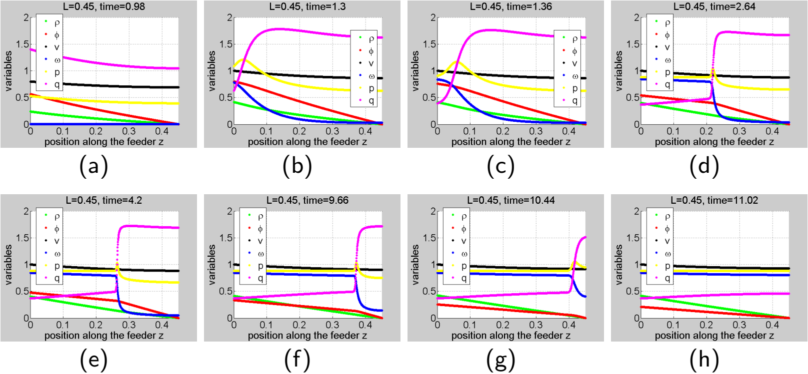
(c)

- (a) Pre fault
- (b) Past voltage drop at the header. Leads to a fully stalled phase.
- (c) Fault is cleared. Front of recovery is advancing towards the tail.

▸ Fault Dynamics

Dynamics/Transitions in Distributed Feeder (IV)

From Stalled to Normal (Movie Recovery)



Of interest: "Soliton"-like shape; voltage profile is (almost) frozen

Fault Dynamics

What can one do at the distribution level to mitigate FIDVR?

- Monitor/learn/model distributed motor parameters
- Control voltage at the head of the line (rise it when needed)
- Distributed reactive control

Why should System Operator worry about FIDVR?

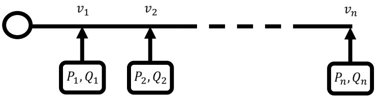
- Simple restoration of the transmission network may not drive the circuits back to a running state.
- A transmission fault $\Rightarrow$ correlated dynamical response in multiple distribution feeders $\Rightarrow$ individual circuits stalled. Specific to each circuit, there is an energy barrier to the transition back to a running state.
- Once a spatially-correlated stalled state exists, the state of the transmission grid has now fundamentally changed.

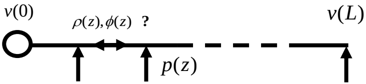
What can the system operator do about FIDVR and related?

- Consider FIDVR as yet another (and much less analyzed !!) transient stability issue/contingency
- Attempt to predict (monitoring short voltage faults within the transmission) ... and pull it back to normal without relying (or with minimal reliance) on the distribution level protection and response

Feeder with Distributed Generation

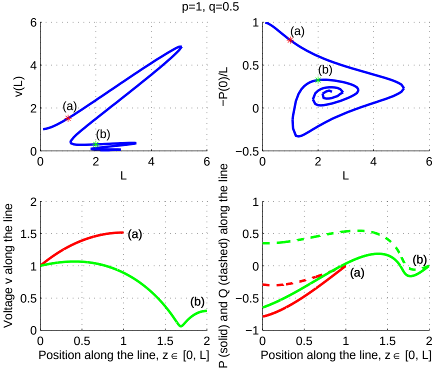
D. Wang, K. Turitsyn, MC (2012)





- Smart Grid Scenario: Significant Penetration of Photo-Voltaic Systems
- Many Consumers feed back to the system
- (Normally) voltage raises down the feeder and feeder exports, $\rho(0), \eta(0) < 0$
- And what if a fault occurs?

Effect of Distributed Generation



The effect is DISTRIBUTED!
not seen in the two-node model

- PV systems inject both p and q
- New regulations will require ride-through-low-voltage capability
- If the distributed generation is too large, multiple low-voltage states will appear
- Prediction of a potential trouble: after a fault the system may be trapped in the low-voltage state (similar to FIDVR)
- The only (normal) way to get out of the trouble is to disconnect the PVs