



## Stability and power sharing in microgrids

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Distributed voltage control

Conclusions and outlook



### Outline



- 2 Microgrids: concept and modeling
- 3 Stability & power sharing with droop control
- 4 Distributed voltage control (DVC)
- 5 Conclusions and outlook

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### Renewables change power system structure



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### Renewables change power system structure



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## Need change in power system operation

- Increasing amount of renewable DG units
- $\Rightarrow$  highly affects in-feed structure of existing power systems
- Most renewable DG units interfaced to network via AC inverters
- Physical characteristics of inverters largely differ from characteristics of SGs
- $\Rightarrow$  Different control and operation strategies are needed



Source: siemens.com

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### The microgrid concept



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# Modeling of microgrids

#### Main network components

- DG units interfaced to network via inverters or SGs
- Loads
- Power lines and transformers

Standard modeling assumptions

- Loads can be modeled by impedances
- Line dynamics can be neglected
- Lossless admittances
- ⇒ Work with Kron-reduced network
- ⇒ DG unit connected at each node in reduced network

Main focus: inverter-based microgrids

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## Main operation modes of inverters in microgrids

- Grid-feeding mode
- Grid-forming mode 2
- Grid-forming units are essential components in power systems
- Tasks

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- To provide a synchronous frequency
- To provide a certain voltage level at all buses in the network
- ⇔ To provide a stable operating point
- ⇒ Focus on inverters in grid-forming mode

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### Basic functionality of DC-AC voltage inverters



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#### Inverter operated in grid-forming mode



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#### Inverter operated in grid-forming mode



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## Grid-forming inverter as controllable voltage source

Model assumptions:

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- Inverter is operated in grid-forming mode
- Its inner current and voltage controllers track references ideally
- If inverter connects intermittent DG unit to network, it is equipped with some sort of fast-reacting storage



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## Grid-forming inverter as controllable voltage source

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# Inverter dynamics

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Inverter model

$$\begin{split} \dot{\delta}_{i} &= u_{i}^{\delta}, \\ \tau_{\mathcal{P}_{i}}\dot{\mathcal{P}}_{i}^{m} &= -\mathcal{P}_{i}^{m} + \mathcal{P}_{i}, \\ V_{i} &= u_{i}^{V}, \\ \tau_{\mathcal{P}_{i}}\dot{\mathcal{Q}}_{i}^{m} &= -\mathcal{Q}_{i}^{m} + \mathcal{Q}_{i} \end{split}$$

Power flows at *i*-th node

$$\begin{split} P_i &= \sum_{k \sim \mathcal{N}_i} |B_{ik}| \sin(\delta_{ik}) V_i V_k \\ Q_i &= |B_{ii}| V_i^2 - \sum_{k \sim \mathcal{N}_i} |B_{ik}| \cos(\delta_{ik}) V_i V_k \end{split}$$

$\delta_i$ $V_i$	phase angle voltage magnitude
$u_i^\delta u_i^V u_i^V$	control inputs
P <sub>i</sub> Q <sub>i</sub>	active power reactive power
$P_i^m$ $Q_i^m$	measured active power measured reactive power
τ <sub>Pj</sub>	time constant of meas. filter

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### Example network

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Inverters in grid-forming mode represented by  $\Sigma_i$ , i = 1, ..., 7

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## Power sharing in microgrids

#### **Definition (Power sharing)**

- Consider an AC electrical network, e.g. an AC microgrid
- Denote its set of nodes by  $\mathcal{N} = [1, n] \cap \mathbb{N}$
- Choose positive real constants  $\gamma_i$ ,  $\gamma_k$ ,  $\chi_i$  and  $\chi_k$
- Proportional active, respectively reactive, power sharing between units at nodes *i* ∈ *N* and *k* ∈ *N*, if

$$\frac{P_i^s}{\gamma_i} = \frac{P_k^s}{\gamma_k}, \quad \text{respectively} \quad \frac{Q_i^s}{\chi_i} = \frac{Q_k^s}{\chi_k}$$

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## Power sharing is an agreement problem

•  $N \subseteq \mathcal{N}$ 

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- $U = \text{diag}(1/\gamma_i), i \in N$
- $D = \operatorname{diag}(1/\chi_i), \quad i \in N$
- Control objective

$$\begin{split} &\lim_{t \to \infty} U P_N(\delta, V) = \upsilon \underline{1}_{|N|}, \\ &\lim_{t \to \infty} D Q_N(\delta, V) = \beta \underline{1}_{|N|}, \quad \upsilon \in \mathbb{R}_{>0}, \quad \beta \in \mathbb{R}_{>0} \end{split}$$

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## Motivation for droop control of inverters

- Droop control: widely used in SG-based power systems to address problems of frequency stability and active power sharing
- ⇒ Adapt droop control to inverters
- ⇒ Make inverters mimic behavior of SGs with respect to frequency and active power
- <sup>(2)</sup> How to couple actuactor signals ( $\dot{\delta}$  and V) with powers (P and Q) to achieve power sharing?
- ⇒ Pose MIMO control design problem as set of decoupled SISO control design problems
- $\Rightarrow$  Analyze couplings in power flow equations over a power line

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## Power flows over a power line

#### Assumptions

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- Dominantly inductive power line with admittance Y<sub>ik</sub> ∈ C between nodes i and k
- Small phase angle differences, i.e.  $\delta_i \delta_k \approx 0$
- ⇒ Approximations

 $Y_{ik} = G_{ik} + jB_{ik} \approx jB_{ik}, \quad \sin(\delta_{ik}) \approx \delta_{ik}, \quad \cos(\delta_{ik}) \approx 1$ 

 $\Rightarrow$  Active and reactive power flows simplify to

$$P_{ik} = -B_{ik} V_i V_k \delta_{ik},$$
  

$$Q_{ik} = -B_{ik} V_i^2 + B_{ik} V_i V_k = -B_{ik} V_i (V_i - V_k)$$

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### Standard droop control for inverters

Frequency droop control

$$u_i^{\delta} = \omega^d - k_{P_i}(P_i^m - P_i^d)$$

Voltage droop control

$$u_i^V = V_i^d - k_{Q_i}(Q_i^m - Q_i^d)$$

Further details: see, e.g., Chandorkar et al. (1993)

$k_{P_i} \in \mathbb{R}_{>0}$	frequency droop
	gain
$\omega^d \in \mathbb{R}_{>0}$	desired (nominal)
	frequency
$P_i^d \in \mathbb{R}$	active power
	setpoint
$P_i^m \in \mathbb{R}$	active power
	measurement
$k_{Q_i} \in \mathbb{R}_{>0}$	voltage droop
	gain
$V_i^d \in \mathbb{R}_{>0}$	desired (nominal)
	voltage amplitude
$Q_i^d \in \mathbb{R}$	reactive power
-	setpoint
$Q^m_i \in \mathbb{R}$	reactive power
	measurement

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#### Droop control - schematic representation



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### Closed-loop droop-controlled microgrid

$$\begin{split} \dot{\delta} &= \omega, \\ T\dot{\omega} &= -\omega + \underline{1}_n \omega^d - K_P (P - P^d), \\ T\dot{V} &= -V + V^d - K_Q (Q - Q^d) \end{split}$$

 $\delta = \operatorname{col}(\delta_i) \in \mathbb{R}^n$  $\omega = \operatorname{col}(\omega_i) \in \mathbb{R}^n$  $V = \operatorname{col}(V_i) \in \mathbb{R}^n$  $V^d = \operatorname{col}(V_i^d) \in \mathbb{R}^n$  $T = \operatorname{diag}(\tau_{P_i}) \in \mathbb{R}^{n \times n}$  $K_P = \operatorname{diag}(k_{P_i}) \in \mathbb{R}^{n \times n}$  $K_Q = \operatorname{diag}(k_{Q_i}) \in \mathbb{R}^{n \times n}$  $P = \operatorname{col}(P_i) \in \mathbb{R}^n$  $Q = \operatorname{col}(Q_i) \in \mathbb{R}^n$  $P^d = \operatorname{col}(P_i^d) \in \mathbb{R}^n$  $Q^d = \operatorname{col}(Q^d_i) \in \mathbb{R}^n$ 

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### Closed-loop droop-controlled microgrid

$$\begin{split} \dot{\delta}_{i} &= \omega^{d} - k_{P_{i}}(P_{i}^{m} - P_{i}^{d}), \\ \tau_{P_{i}}\dot{P}_{i}^{m} &= -P_{i}^{m} + P_{i}, \\ V_{i} &= V_{i}^{d} - k_{Q_{i}}(Q_{i}^{m} - Q_{i}^{d}), \\ \tau_{P_{i}}\dot{Q}_{i}^{m} &= -Q_{i}^{m} + Q_{i} \\ & \downarrow \\ \text{change of variables} \\ & \downarrow \\ \text{change of variables} \\ & \downarrow \\ \dot{\delta} &= \omega, \\ T\dot{\omega} &= -\omega + \underline{1}_{n}\omega^{d} - K_{P}(P - P^{d}) \\ T\dot{V} &= -V + V^{d} - K_{Q}(Q - Q^{d}) \end{split}$$

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 $\delta = \operatorname{col}(\delta_i) \in \mathbb{R}^n$  $\omega = \operatorname{col}(\omega_i) \in \mathbb{R}^n$  $V = \operatorname{col}(V_i) \in \mathbb{R}^n$  $V^d = \operatorname{col}(V_i^d) \in \mathbb{R}^n$  $T = \operatorname{diag}(\tau_{P_i}) \in \mathbb{R}^{n \times n}$  $K_P = \operatorname{diag}(k_{P_i}) \in \mathbb{R}^{n \times n}$  $K_Q = \operatorname{diag}(k_{Q_i}) \in \mathbb{R}^{n \times n}$  $P = \operatorname{col}(P_i) \in \mathbb{R}^n$  $Q = \operatorname{col}(Q_i) \in \mathbb{R}^n$  $P^d = \operatorname{col}(P_i^d) \in \mathbb{R}^n$  $Q^d = \operatorname{col}(Q^d_i) \in \mathbb{R}^n$ 

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### **Problem statement**

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- Derive conditions for asymptotic stability of generic droop-controlled microgrids
- Investigate if droop control is suitable to achieve control objective of active power sharing, i.e.,

$$\frac{P_i^s}{\gamma_i} = \frac{P_k^s}{\gamma_k}, \quad \gamma_i \in \mathbb{R}_{>0}, \quad \gamma_k \in \mathbb{R}_{>0}$$

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## Stability analysis

- Coordinate transformation
- Follow interconnection and damping assignment passivity-based control (IDA-PBC) approach (Ortega et al. (2002))
- Represent microgrid dynamics in port-Hamiltonian form
- Can easily identify energy function

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### Port-Hamiltonian systems

$$\begin{split} \dot{x} &= (J(x) - R(x)) \, \nabla H + g(x) u, \quad x \in \mathbb{X} \subseteq \mathbb{R}^n, \quad u \in \mathbb{R}^m, \\ y &= g^T(x) \nabla H, \quad y \in \mathbb{R}^m \end{split}$$

- $J(x) \in \mathbb{R}^{n \times n}$ ,  $J(x) = -J(x)^T$  (interconnection matrix)
- $R(x) \ge 0 \in \mathbb{R}^{n \times n}$  for all  $x \in \mathbb{X}$  (damping matrix)
- $H: \mathbb{X} \to \mathbb{R}$  (Hamiltonian),  $\nabla H = \left(\frac{\partial H}{\partial x}\right)^T$
- Power balance equation

$$\underbrace{\dot{H}}_{\text{stored power}} = -\underbrace{\nabla H^T R(x) \nabla H}_{\text{dissipated power}} + \underbrace{u^T y}_{\text{supplied power}} \leq u^T y$$

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### Synchronized motion

• Synchronized motion starting in  $(\delta^{s}, \underline{1}_{n}\omega^{s}, V^{s}) \in \mathbb{S}^{n} \times \mathbb{R}^{n} \times \mathbb{R}^{n}_{>0}$  $\delta^{*}(t) = \operatorname{mod}_{2\pi} \left\{ \delta^{s} + \underline{1}_{n}\omega^{s}t \right\},$  $\omega^{*}(t) = \underline{1}_{n}\omega^{s},$ 

 Aim: derive conditions, under which solutions of microgrid converge asymptotically to synchronized motion

 $V^*(t) = V^s$ 

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### Power flows depend on angle differences $\delta_{ik}$

$$P_i(\delta_1,\ldots,\delta_n,V_1,\ldots,V_n)=\sum_{k\sim\mathcal{N}_i}|B_{ik}|\sin(\delta_{ik})V_iV_k,$$

$$Q_i(\delta_1,\ldots,\delta_n,V_1,\ldots,V_n) = |B_{ii}|V_i^2 - \sum_{k\sim\mathcal{N}_i} |B_{ik}|\cos(\delta_{ik})V_iV_k$$

- ⇒ Flow of system can be described in reduced angle coordinates
- $\Rightarrow$  Transform convergence problem into classical stability problem

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## Main result (1) - Stability

#### Proposition (A condition for local asymptotic stability)

- Fix  $\tau_{P_i}$ ,  $k_{P_i}$ ,  $\omega^d$  and  $P_i^d$
- If  $V_i^d$ ,  $k_{Q_i}$  and  $Q_i^d$  are chosen such that

$$\mathcal{D} + \mathcal{T} - \mathcal{W}^{\top} \mathcal{L}^{-1} \mathcal{W} > 0 \tag{1}$$

⇒ equilibrium point is locally asymptotically stable

$$\mathcal{L} > 0, \quad \mathcal{T} > 0, \quad \mathcal{D} = diag\left(rac{V_i^d + k_{Q_i}Q_i^d}{k_{Q_i}(V_i^s)^2}
ight) > 0$$

Condition (1) ensures that Hamiltonian is locally positive definite

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# Main result (2) - Active power sharing

Lemma (A condition for active power sharing)

- Assume microgrid possesses synchronized motion
- Then all generation units share active power proportionally with respect to γ<sub>i</sub> and γ<sub>k</sub> in steady-state if

$$k_{P_i}\gamma_i = k_{P_k}\gamma_k$$
 and  $\frac{P_i^d}{\gamma_i} = \frac{P_k^d}{\gamma_k}$ ,  $i \sim \mathcal{N}$ ,  $k \sim \mathcal{N}$ 

Condition holds independently of line admittances

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## CIGRE MV benchmark model



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### Simulation example

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### Reactive power sharing

- Voltage droop control does, in general, not guarantee desired reactive power sharing
- ⇒ Several other or modified (heuristic) decentralized voltage control strategies proposed in literature (Zhong (2013), Li et al. (2009), Sao et al. (2005), Simpson-Porco et al. (2014),...)
  - But: no general conditions or formal guarantees for reactive power sharing are given

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### Inverter model revisited

Inverter dynamics

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$$\begin{split} \dot{\delta}_{i} &= u_{i}^{\delta}, \\ \tau_{P_{i}} \dot{P}_{i}^{m} &= -P_{i}^{m} + P_{i}, \\ V_{i} &= u_{i}^{V}, \\ \tau_{P_{i}} \dot{Q}_{i}^{m} &= -Q_{i}^{m} + Q_{i} \end{split}$$

Reactive power flow at *i*-th node

$$Q_i(\delta_1,\ldots,\delta_n,V_1,\ldots,V_n) = |B_{ii}|V_i^2 - \sum_{k \sim \mathcal{N}_i} |B_{ik}|\cos(\delta_{ik})V_iV_k$$

$\delta_i$ $V_i$	phase angle voltage magnitude
$u_i^\delta u_i^V$	control inputs
P <sub>i</sub> Q <sub>i</sub>	active power reactive power
$P_i^m$ $Q_i^m$	measured active power measured reactive power
$\tau_{P_i}$	time constant of meas. filter

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### Inverter model revisited

Inverter dynamics

$$\begin{split} \dot{\delta}_{i} &= u_{i}^{\delta}, \\ \tau_{P_{i}} \dot{P}_{i}^{m} &= -P_{i}^{m} + P_{i} \\ V_{i} &= u_{i}^{V}, \\ \tau_{P_{i}} \dot{Q}_{i}^{m} &= -Q_{i}^{m} + Q_{i} \end{split}$$

Reactive power flow at *i*-th node with  $\delta_{ik} \approx 0$ 

$$Q_i(V_1,\ldots,V_n) = |B_{ii}|V_i^2 - \sum_{k \sim \mathcal{N}_i} |B_{ik}|V_iV_k$$

phase angle  $\delta_i$ Vi voltage magnitude  $u_i^{\delta}$  $u_i^{V}$ control inputs  $P_i$ active power  $Q_i$ reactive power  $P_i^m$ measured active power  $Q_i^m$ measured reactive power time constant  $\tau_{P_i}$ of meas. filter



#### Problem

Design a voltage control law such that

- the microgrid possesses an asymptotically stable equilibrium point
- 2 the DG units share their reactive powers proportionally in steady-state

$$\Leftrightarrow \lim_{t\to\infty} DQ(V) = \beta \underline{1}_n, \quad D = diag(1/\chi_i) \in \mathbb{R}^{n \times n}, \quad \beta \in \mathbb{R}$$



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### Consensus-based distributed voltage control (DVC)

$$u_i^V = V_i^d - k_i \int_0^t e_i(\tau) d\tau,$$
$$e_i = \sum_{k \sim C_i} \left( \frac{Q_i^m}{\chi_i} - \frac{Q_k^m}{\chi_k} \right)$$
$$= \sum_{k \sim C_i} (\bar{Q}_i - \bar{Q}_k)$$

- $V_i^{\mathsf{d}} \in \mathbb{R}_{>0}$  desired (nominal) voltage magnitude
- $k_i \in \mathbb{R}_{>0}$  feedback gain

 $\mathcal{C}_i$ 

set of neighbor nodes of node *i* in graph induced by communication network

 Assumption: graph induced by communication network is connected and undirected

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#### Consensus-based distributed voltage control (DVC)



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#### Closed-loop voltage and reactive power dynamics

 $\mathcal{L} \in \mathbb{R}^{n \times n} \dots$  Laplacian matrix of connected undirected graph induced by communication network

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#### Closed-loop voltage and reactive power dynamics

 $\mathcal{L} \in \mathbb{R}^{n \times n} \dots$  Laplacian matrix of connected undirected graph induced by communication network



#### Conclusions and outlook



#### Claim

The DVC achieves proportional reactive power sharing in steady-state.

#### Sketch of proof

- L... Laplacian matrix of connected undirected graph
- $\Rightarrow \mathcal{L} = \mathcal{L}^T \ge 0, \quad \mathcal{L}\underline{1}_n = \underline{0}_n, \quad v^T \mathcal{L} v > 0 \text{ for all } v \in \mathbb{R} \setminus \{\beta \underline{1}_n\}, \beta \in \mathbb{R}$

#### Steady-state

$$\dot{V} = \underline{0}_n = -K\mathcal{L}DQ^s \Leftrightarrow DQ^s = \beta \underline{1}_n \Leftrightarrow \frac{Q_i^s}{\chi_i} = \frac{Q_k^s}{\chi_k}$$

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#### Voltage conservation law

• 
$$\underline{1}_n^T \mathcal{L} = \underline{0}_n^T$$

$$\Rightarrow \underline{1}_n^T K^{-1} \dot{V} = \underline{1}_n^T K^{-1} K \mathcal{L} D Q^m = \underline{0}_n^T D Q^m$$

$$\Leftrightarrow \sum_{i=1}^{n} \frac{\dot{V}_i}{k_i} = 0$$

⇒ Describe flow of system in reduced voltage coordinates for stability analysis

$$V_R = \operatorname{col}(V_i) \in \mathbb{R}_{>0}^{n-1},$$
  
$$V_n = V_n(V_R) = \sum_{i=1}^n \frac{V_i(0)}{k_i} - \sum_{i=1}^{n-1} \frac{k_n}{k_i} V_i$$

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## for local exponential stability

### Proposition

Motivation

- Fix D and a positive real constant au
- Set  $\tau_{P_i} = \tau$ ,  $i \sim \mathcal{N}$  and  $K = \kappa D$ ,  $\kappa \in \mathbb{R}_{>0}$
- Let  $N = \frac{\partial Q}{\partial V}|_{x^{S}}$
- Let µ<sub>i</sub> = a<sub>i</sub> + jb<sub>i</sub> be the i-th nonzero eigenvalue of the matrix product NDLD with a<sub>i</sub> ∈ ℝ and b<sub>i</sub> ∈ ℝ
- Then  $\mu_i \in \mathbb{C}^+$
- Furthermore, x<sup>s</sup> is a locally exponentially stable equilibrium point if and only if the positive real parameter κ is chosen such that

$$au\kappa b_i^2 < a_i$$

for all  $\mu_i$ 

 Moreover, x<sup>s</sup> is locally exponentially stable for any positive real κ if and only if NDLD has only real eigenvalues



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### Subnetwork 1 of CIGRE MV benchmark model



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#### Simulation example



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### Conclusions and outlook

Microgrids

- Microgrids are a promising concept in networks with large amount of DG
- Condition for local asymptotic stability in lossless droop-controlled inverter-based microgrids
- Selection criterion for parameters of frequency droop control that ensures desired active power sharing in steady-state
- Proposed distributed voltage control (DVC), which solves open problem of reactive power sharing

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### Outlook and related work

Microgrids

- Analysis of microgrids with frequency droop control and distributed voltage control (DVC)
- Control schemes for highly ohmic networks
- Influence of clock drift and delay induced by digital controllers of inverters on microgrid performance (submitted to ACC'15)
- Secondary frequency control (Simpson-Porco et al. (2013), Bürger et al. (2014), Andreasson et al. (2012), Bidram et al. (2013), Shafiee et al. (2014))
- Optimal operation control (Dörfler et al. (2014), Bolognani et al. (2013), Hans et al.(2014))
- Alternative inverter control schemes (Zhong et al. (2011), Torres et al. (2014), Dhople et al. (2014))

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#### **Publications**



Schiffer, J., Ortega, R., Astolfi, A., Raisch, J. and Sezi, T.

Conditions for Stability of Droop-Controlled Inverter-Based Microgrids, Automatica, In Press, 2014



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Schiffer, J., Seel, T., Raisch, J. and Sezi, T.,

Voltage Stability and Reactive Power Sharing in Inverter-Based Microgrids with Consensus-Based Distributed Voltage Control Submitted to IEEE Transactions on Control Systems Technology, 2014



Schiffer, J., Seel, T., Raisch, J. and Sezi, T.,

A Consensus-Based Distributed Voltage Control for Reactive Power Sharing in Microgrids 13th ECC, Strasbourg, France, 2014



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