Hierarchical MPC control for Plug-and-Play resource distribution

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Outline

1. Problem formulation

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2. Proposed solution

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Distributed resource control

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- The system has a number of decentral storages that can each store a certain amount of some resource
- Each storage can be filled or emptied at some maximal rate(s)
- A central controller has the responsibility of balancing supply and demand by use of the storages

Problem formulation

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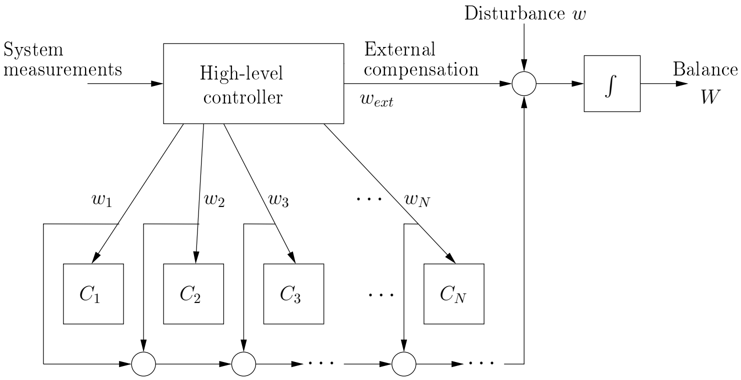
- *The solution is scalable*
- *The solution supports plug-and-play of subsystems*

Problem setup



High-level controller

The high-level controller must solve the following (discrete-time) optimisation problem at any given time t:

$$
\begin{aligned}
\min_{w_i, w_{\text{ext}}} \quad & \sum_{k=1}^{N_h} \rho W_k^2 + \phi(w_{\text{ext},k}, w_{\text{ext},k-1}) \\
\text{s.t.} \quad & \underline{W} \leq W_k \leq \bar{W} \\
& \underline{w}_i \leq w_{i,k} \leq \bar{w}_i, 1 \leq i \leq N
\end{aligned}
$$

$\underline{W}$, $\bar{W}$: Constraints on the balance,

ρ: Scalar cost on balance

$\phi : \mathbb{R} \times \mathbb{R} \to \mathbb{R}_+$: A cost function of the absolute value of w_{ext} as well as changes in w_{ext}.

N_h: is the prediction horizon of the controller.

Storage model

Consumption by the storages are for simplicity assumed cost-free. Furthermore, it is assumed that each storage can be modelled as a marginally stable linear systems.
Each storage is characterized by its own state equation

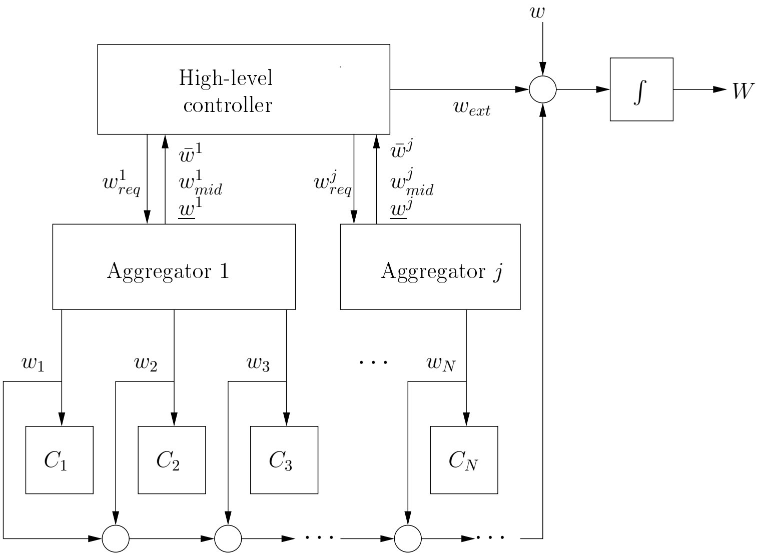
$$\frac{dW_i(t)}{dt} = w_i(t)$$

which must satisfy $0 \leq W_i(t) \leq \bar{W}_i$ at all times.

Furthermore, each storage has limits on how much of the resource it will accept at any given time, $\underline{w}_i \leq w_i(t) \leq \bar{w}_i$.

Outline

Modified architecture



Resource balance

The main objective of the high-level control is to keep the resource balance governed by

$$\frac{dW(t)}{dt} = w_{\text{ext}}(t) - w(t) - w_a(t)$$

at zero.

$w(t)$: An external disturbance

$w_a(t) = \sum_i^N w_i(t)$: The resource absorbed by the intelligent storages $(C_i, 1 \leq i \leq N)$.

It is assumed that the top level controller can control $w_{\text{ext}}(t)$ directly and constrained only by a rate limit, but we would also like to keep the time derivative small.

The aggregator

At each sample, the aggregator solves the simple optimisation problem of

$$\min_{w_i} \sum (W_i(t+T_s) - W_{i,\mathrm{ref}})^2,$$

s.t.

$$\begin{array}{l} \sum_i w_i = w_{\mathrm{req}}, \\ \underline{w}_i \le w_i(t) \le \bar{w}_i \\ 0 \le W_i(t+T_s) \le \bar{W}_i \end{array}$$

with $W_i(t+T_s) = W_i(t) + T_s w_i$, where T_s is the sampling time.

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- The deviation from the nominal consumption is minimized for each storage
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- The rate constraint for each storage is not violated

Stability

First, in order to assess stability, it is of course necessary to establish a sensible definition, especially as the system has constraints with the usual implications for stability.
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Intuitively, stability for a system of the type described above will comprise the following features:

1. For constant inputs, all trajectories will tend to constant values
2. In steady state, a minimal number of constraints will be invoked
3. Wind-up behavior of system states is avoided for any bounded input set

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- . . . it should be expected that the complexity could be further reduced by implementing a dedicated quadratic programming solver, which exploits the sparsity.
- If the quadratic cost function is not central, a linear cost can be applied at the aggregator level, making the complexity in no. of storages linear.

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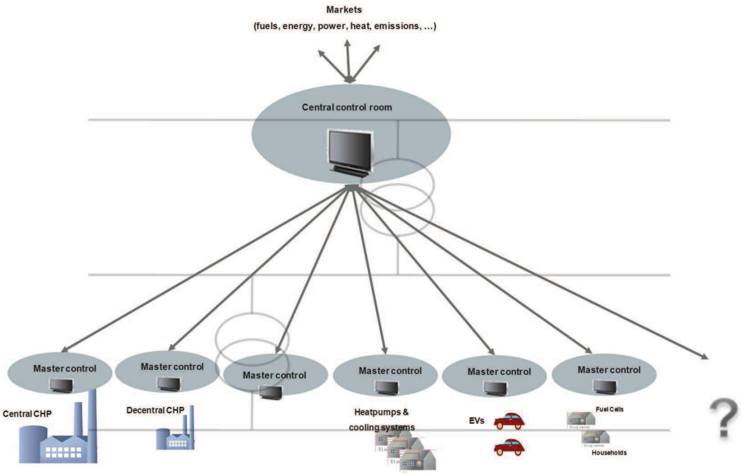
Virtual Power Plants



Markets
(fuels, energy, power, heat, emissions, ...)

Central control room

Master control

Master control

Master control

Master control

Master control

Master control

Central CHP

Decentral CHP

Heatpumps & cooling systems

EVs

Fuel Cells

Households

?

Example: Smart Grid application

The main objective of the top level control is to keep the energy balance governed by

$$\frac{dE(t)}{dt} = P_{\text{ext}}(t) - P_{\text{load}}(t) - P_{\text{a}}(t) \tag{1}$$

at zero.

$P_a = \sum_i P_i$: Power absorbed by the intelligent consumers (ICs).

P_{load}: Power absorbed by other consumers (considered as a disturbance here).

P_{ext}: Power produced by a number of suppliers such as power plants etc.

It is assumed that the top level controller can control P_{ext} directly and restrained only by a rate limit, but we would also like to keep the time derivative small.

Example: Smart Grid application

Each intelligent consumer is characterized by its own energy balance

$$\frac{dE_i(t)}{dt} = P_i(t)$$

which must satisfy $0 \leq E_i(t) \leq \bar{E}_i$ at all times. Furthermore, each intelligent consumer can only consume a limited amount of power $\underline{P}_i \leq P_i(t) \leq \bar{P}_i$.

Optimization problem

At each sample, at time t, the aggregator solves the simple optimisation problem of

$$\begin{array}{l}\min_{P_i} \sum (E_i(t+T_s) - E_{i,\mathrm{ref}})^2, \\ \text{s.t.} \\ \sum P_i = P_{\mathrm{req}}, \\ \underline{P}_i \leq P_i(t) \leq \bar{P}_i, \\ 0 \leq E_i(t+T_s) \leq \bar{E}_i \end{array}$$

with $E_i(t+T_s) = E_i(t) + T_s P_i$, thereby distributing the power in a way that brings the energy levels as close to the reference as possible in a quadratic sense.

High level optimization

The top level control optimizes over a prediction horizon N_p. It minimizes the performance function

$$
\begin{aligned}
J_t = & \sum_{i=1}^{N_p} E(t + T_s i)^2 \\
& + \beta_p \sum_{i=1}^{N_c} (P_{\text{ext}}(t + T_s i) - P_{\text{ext}}(t + T_s(i-1)))^2 \\
& + \beta_r \sum_{i=1}^{N_c} (P_{\text{req}}(t + T_s i) - P_{\text{mid}}(t))^2
\end{aligned}
$$

with N_c samples of P_{ext} and P_{req} as decision variables.

Constraints

The optimisation is subject to constraints on the decision variables. There is a rate limit on the power from the power plants:

$$\underline{P}_{\mathrm{ext}} \leq P_{\mathrm{ext}}(t + T_s i) - P_{\mathrm{ext}}(t + T_s(i-1)) \leq \bar{P}_{\mathrm{ext}}$$

The aggregator provides limits on P_a that can be sustained over a horizon N_l. These limits are conservative in the sense that if P_{req} is for instance negative for the first part of the horizon, then a positive P_{req} higher than $\bar{P}$ may be feasible for the rest. However, in order to simplify the top level computations, the constraint

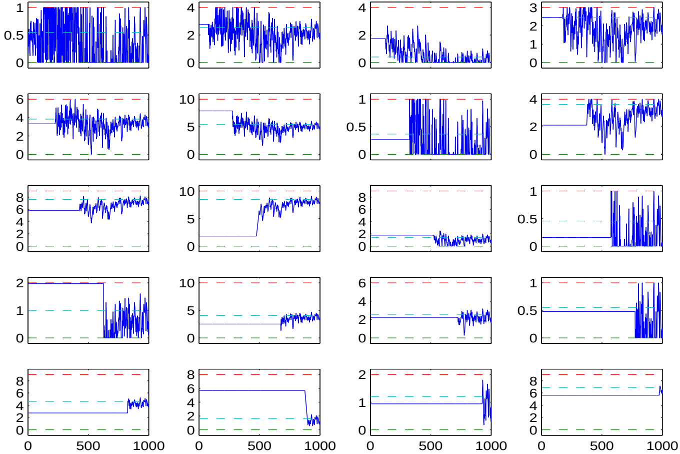
$$\underline{P}(t) \leq P_{\mathrm{req}}(t + T_s i) \leq \bar{P}(t)$$

is imposed over the whole horizon.

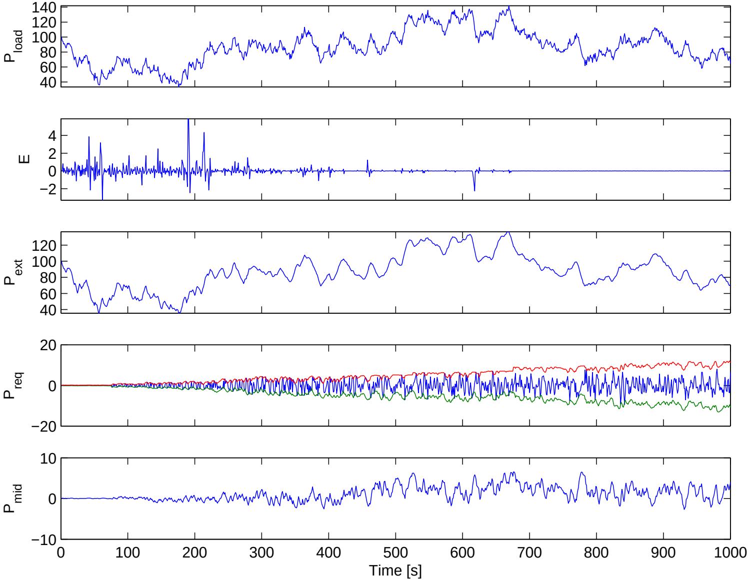
Parameters for 20 consumers in simulation

i	$\bar{E}_i$	$\underline{P}_i$	$\bar{P}_i$	i	$\bar{E}_i$	$\underline{P}_i$	$\bar{P}_i$
1	1.0	-1.7	1.4	11	9.0	-0.2	1.1
2	4.0	-1.4	0.8	12	1.0	-1.0	1.2
3	4.0	-0.2	1.8	13	2.0	-1.6	1.6
4	3.0	-1.3	0.3	14	10.0	-1.3	1.9
5	6.0	-1.6	0.9	15	6.0	-0.3	0.4
6	10.0	-1.3	1.1	16	1.0	-1.1	0.9
7	1.0	-0.7	1.2	17	9.0	-1.9	1.8
8	4.0	-1.9	0.2	18	8.0	-0.2	0.8
9	9.0	-1.1	0.2	19	2.0	-0.9	0.6
10	10.0	-1.1	0.2	20	9.0	-1.6	0.5

Simulation with aggregator



Simulation example



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- The proposed solution is scalable and can be implemented with a linear growth in complexity.
- The proposed solution admits plug-and-play of new units simply by adding them in the aggregator tables.
- The approach is sensitive to prediction horizon. This is not surprising by emphasizes the need for good load predictions.