Cooperation-based optimization of industrial supply chains

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May 19–21, 2010 Workshop on Distributed Model Predictive Control and Supply Chains Lund Center for Control of Complex Engineering Systems Lund University

Rawlings	Opt

Optimization of supply chains

Overview of Distributed Model Predictive Control

- Control of large-scale systems
- Stability theory for cooperative MPC

2 Challenges for Cooperative MPC of Supply Chains



Conclusions and Future Outlook

Predictive control



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Optimization of supply chains

State estimation



Electrical power distribution



Chemical plant integration





Decentralized Control

- Most large-scale systems consist of networks of interconnected/interacting subsystems
 - Chemical plants, electrical power grids, water distribution networks, ...

Decentralized Control

- Most large-scale systems consist of networks of interconnected/interacting subsystems
 - Chemical plants, electrical power grids, water distribution networks, ...
- Traditional approach: Decentralized control
 - Wealth of literature from the early 1970's on improved decentralized control ^a
 - Well known that poor performance may result if the interconnections are not negligible

^a(Sandell Jr. et al., 1978; Šiljak, 1991; Lunze, 1992)

Centralized Control

- Steady increase in available computing power has provided the opportunity for centralized control
- Coordinated control: Distributed optimization to achieve fast solution of centralized control (Necoara et al., 2008; Cheng et al., 2007)
- Most practitioners view centralized control of large, networked systems as impractical and unrealistic
- A divide and conquer strategy is essential for control of large, networked systems (Ho, 2005)
- Centralized control: A benchmark for comparing and assessing distributed controllers

Objective functions	$V_1(u_1, u_2), \ V_2(u_1, u_2)$
and	$V(u_1, u_2) = w_1 V_1(u_1, u_2) + w_2 V_2(u_1, u_2)$
decision variables for units	$u_1\in\Omega_1, u_2\in\Omega_2$

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Cooperative Control	$\min_{u_1\in\Omega_1}V(u_1,u_2)$	$\min_{u_2\in\Omega_2}V(u_1,u_2)$
(Pareto optimal)		
Centralized Control	$\min_{\substack{u_1,u_2\in\Omega_1\times\Omega_2}}$	$V(u_1, u_2)$
(Pareto optimal)		

Noninteracting systems



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Weakly interacting systems



Moderately interacting systems



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Optimization of supply chains

Strongly interacting (conflicting) systems



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Strongly interacting (conflicting) systems



 u_2

Geometry of cooperative vs. noncooperative MPC



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Optimization of supply chains

Geometry of cooperative vs. noncooperative MPC



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Optimization of supply chains



- Early termination of optimization gives suboptimal plantwide feedback
- Use suboptimal MPC theory to prove stability

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Consider closed-loop system augmented with input trajectory

$$\begin{pmatrix} x^+ \\ \mathbf{u}^+ \end{pmatrix} = \begin{pmatrix} Ax + Bu \\ g(x, \mathbf{u}) \end{pmatrix}$$

• Function $g(\cdot)$ returns suboptimal choice

¹(Rawlings and Mayne, 2009, pp.418-420)

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- Stability of augmented system is established by Lyapunov function

$$\begin{aligned} \mathsf{a} \left| (x, \mathbf{u}) \right|^2 &\leq V(x, \mathbf{u}) \leq \mathsf{b} \left| (x, \mathbf{u}) \right|^2 \\ V(x^+, \mathbf{u}^+) - V(x, \mathbf{u}) \leq -\mathsf{c} \left| (x, u) \right|^2 \end{aligned}$$

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$$|a|(x, \mathbf{u})|^2 \le V(x, \mathbf{u}) \le b|(x, \mathbf{u})|^2$$

 $V(x^+, \mathbf{u}^+) - V(x, \mathbf{u}) \le -c|(x, u)|^2$

 $\bullet\,$ Adding constraint establishes closed-loop stability of the origin for all u^1

$$|\mathbf{u}| \le d |x| \quad x \in \mathbb{B}_r, r > 0$$

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• Cooperative optimization satisfies these properties for plantwide objective function *V*(*x*, **u**)

¹(Rawlings and Mayne, 2009, pp.418-420)

Plantwide step response



²Gudi and Rawlings (2006)

Rawlings

Optimization of supply chains

Plantwide step response



• Interaction models found by decentralized identification²

²Gudi and Rawlings (2006)

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 $x_{11}^{+} = A_{11}x_{11} + B_{11}u_1$ $x_{21}^{+} = A_{21}x_{21} + B_{21}u_1$

Consider the linearized physical model

$$x^+ = Ax + B_1u_1 + B_2u_2$$
 $y_1 = C_1x$, $y_2 = C_2x$

• Kalman canonical form of the triple (A, B_j, C_i)



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Interaction models

$$A_{ij} \leftarrow A^{oc}_{ij} \quad B_{ij} \leftarrow B^{oc}_{ij} \quad C_{ij} \leftarrow C^{oc}_{ij} \quad x_{ij} \leftarrow z^{oc}_{ij}$$

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Unstable modes

For unstable systems, we zero the unstable modes with terminal constraints.

• For subsystem 1

$$S_{11}^{u'}x_{11}(N) = 0$$
 $S_{21}^{u'}x_{21}(N) = 0$

• To ensure terminal constraint feasibility for all *x*, we require (<u>A</u>₁, <u>B</u>₁) stabilizable

$$\underline{A}_1 = \begin{bmatrix} A_{11} & \\ & A_{21} \end{bmatrix} \qquad \underline{B}_1 = \begin{bmatrix} B_{11} \\ & B_{21} \end{bmatrix}$$

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• For output feedback, we require (A_1, C_1) detectable

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• Similar requirements for other subsystem

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Output feedback

Consider augmented system perturbed by stable estimator

$$\begin{pmatrix} \hat{x}^+ \\ \mathbf{u}^+ \\ e^+ \end{pmatrix} = \begin{pmatrix} A\hat{x} + B\mathbf{u} + Le \\ g(\hat{x}, \mathbf{u}, e) \\ A_L e \end{pmatrix}$$

• Stable estimator error implies Lyapunov function

$$egin{array}{l} ar{a} \left| e
ight| \leq \! J(e) \leq ar{b} \left| e
ight| \ J(e^+) \!-\! J(e) \leq -ar{c} \left| e
ight| \end{array}$$

• Stability of perturbed system established by Lyapunov function

$$W(\hat{x},\mathbf{u},e) = V(\hat{x},\mathbf{u}) + J(e)$$

Two reactors with separation and recycle



Two reactors with separation and recycle



Performance comparison

	Cost ($\times 10^{-2}$)	Performance loss
Centralized MPC	1.75	0
Decentralized MPC	∞	∞
Noncooperative MPC	∞	∞
Cooperative MPC (1 iterate)	2.2	25.7%
Cooperative MPC (10 iterates)	1.84	5%

- Previous work on supply chain modeling and optimization³
- Inventories and backorders are subsystem states
- Downstream product shipments and upstream orders are subsystem inputs
- Inventories and backorders modeled as integrators (tanks)
- Stabilizability and detectability assumptions not satisfied

$$\underline{A}_{i} = \begin{bmatrix} I & \\ & I \end{bmatrix} \qquad \underline{B}_{i} = \begin{bmatrix} B_{1i} \\ B_{2i} \end{bmatrix}$$
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• Implementation of cooperative MPC for supply chains remains a challenge

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Possible solution I: Coupled constraints

- Work with minimal (A, B, C) supply chain model
- Terminal constraint $S^{u'}x(N) = 0$ coupled in subsystem inputs

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- Cooperative optimization does not converge to Pareto optimum with coupled constraints
- Share coupled inputs among subsystems to achieve Pareto optimal performance
- In the limit of full supply chain coupling, each subsystem solves the centralized optimization

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Alternative

• To avoid centralized optimization, share inputs with only nearest neighbors for near optimal performance

Possible solution II: Centralized estimation

• $(\underline{A}_i, \underline{B}_i)$ not stabilizable, but there is a stabilizable subspace $\underline{\mathbb{X}}_i$

$$\underline{\mathbb{X}}_{i} = \left\{ \underline{\mathsf{x}}_{i} \mid \exists \mathbf{u}_{i} : \begin{bmatrix} \underline{\mathsf{A}}_{i}^{n-1} \underline{\mathsf{B}}_{i} & \cdots & \underline{\mathsf{B}}_{i} \end{bmatrix} \mathbf{u}_{i} = -\underline{\mathsf{A}}_{i}^{n} \underline{\mathsf{x}}_{i} \right\}$$

• Any $\underline{x}_i \in \underline{\mathbb{X}}_i$ can be brought to the origin

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Trade-offs

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- Easy to enforce $\underline{x}_i \in \underline{\mathbb{X}}_i$

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Trade-offs

- No coupled constraints, therefore cooperative optimization converges to Pareto optimum
- Easy to enforce $\underline{x}_i \in \underline{\mathbb{X}}_i$
- Subsystems must share output measurements
- Supply chain subsystems cannot choose estimators independently

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Conclusions

• Cooperative MPC theory maturing^a

- satisfies hard input constraints
- provides nominal stability for plants with even strongly interacting subsystems
- retains closed-loop stability for early iteration termination
- converges to Pareto optimal control in the limit of iteration
- remains stable under perturbation from stable state estimator
- avoids coordination layer

^aStewart et al. (2010b); Maestre et al. (2010)

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- retains closed-loop stability for early iteration termination
- converges to Pareto optimal control in the limit of iteration
- remains stable under perturbation from stable state estimator
- avoids coordination layer
- Cooperative MPC for supply chains remains a challenge
 - stabilizability and detectability assumptions not satisfied
 - many alternative solution strategies exist
 - each strategy has drawbacks

^aStewart et al. (2010b); Maestre et al. (2010)

Future directions

Supply chains

- Evaluate alternative supply chain cooperative control strategies^a
- Industrial application: gas supplier (Praxair), steel mill, power utility

^aAltmüller et al. (2010); Mårtensson and Rantzer (2009)

Future directions

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Cooperative MPC

- Hierarchical implementation^a
 - time scale separation
 - delayed communication
 - reduced information sharing
 - optimization at MPC layer only
- Nonlinear models

^aStewart et al. (2010a)

MPC Monograph — Chapter 6 on distributed MPC



- 576 page text
- 214 exercises
- 335 page solution manual
- 3 appendices on web (133 pages)
- www.nobhillpublishing.com

Further reading I

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