Plug-and-Play Control and Optimization in Microgrids

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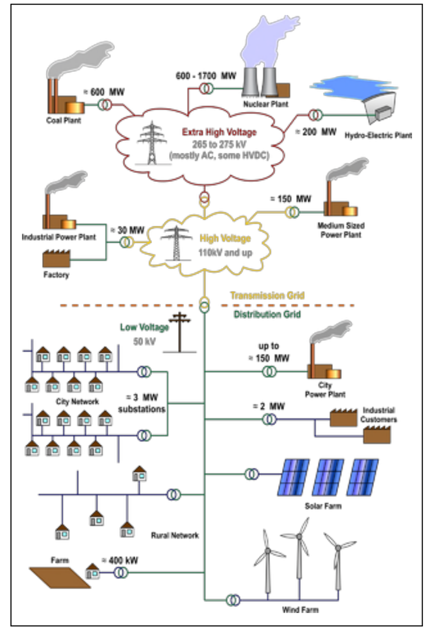
LCCC Dynamics and Control in Networks Workshop

Paradigm shifts in the operation of power networks



Traditional **top to bottom** operation:

- generate/transmit/distribute power
- hierarchical control & operation

Smart & green **power to the people**:

- high renewable penetration
- distributed generation & deregulation
- demand response & load control

Microgrids

Structure

- low-voltage distribution networks
- grid-connected or islanded
- autonomously managed

Applications

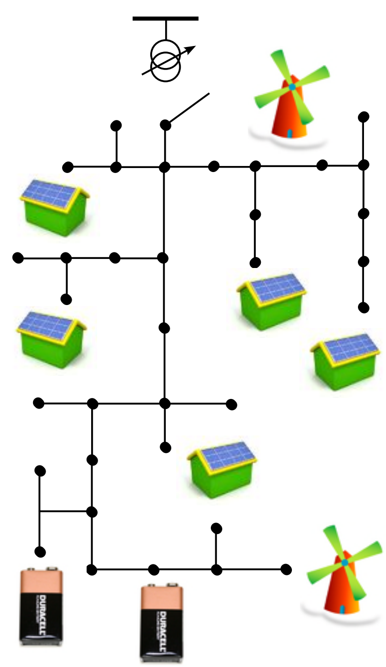
- hospitals, military, campuses, large vehicles, & isolated communities

Benefits

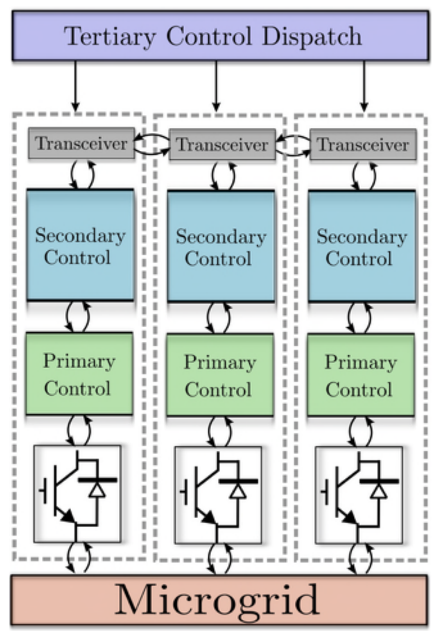
- naturally distributed for renewables
- flexible, efficient, & reliable

Operational challenges

- volatile dynamics & low inertia
- plug'n'play & no central authority



Conventional control architecture from bulk power ntwks



3. **Tertiary control** (offline)
   - Goal: optimize operation
   - Strategy: centralized & forecast

2. **Secondary control** (slower)
   - Goal: maintain operating point
   - Strategy: centralized

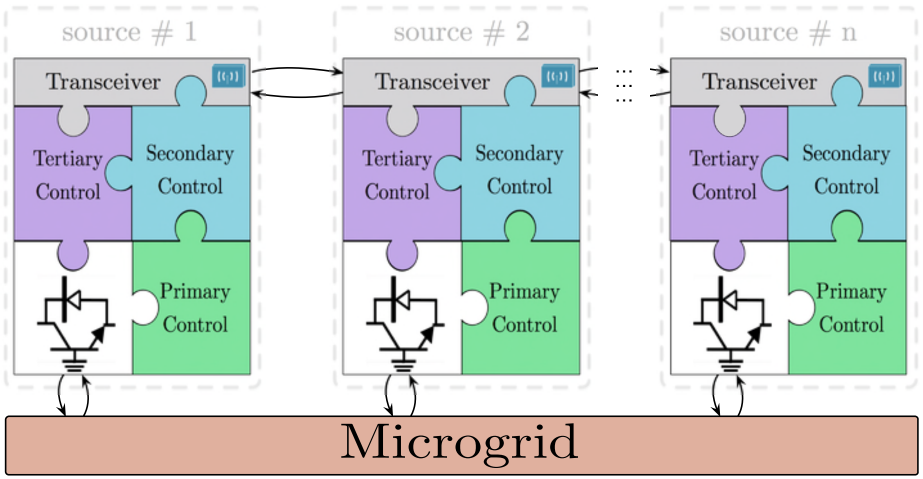
1. **Primary control** (fast)
   - Goal: stabilization & load sharing
   - Strategy: decentralized

Microgrids: distributed, model-free, online & without time-scale separation

⇒ **break** vertical & horizontal **hierarchy**

A preview – plug-and-play control and optimization

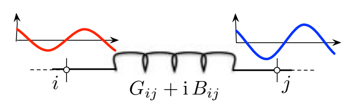
flat hierarchy, distributed, no time-scale separations, & model-free . . .



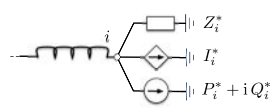
Outline

Modeling: a microgrid is a circuit

1. synchronous (& acyclic) **AC circuit** with harmonic waveforms $E_i e^{i(\theta_i + \omega^* t)}$



2. **ZIP loads**: constant impedance, current, & power $P_i^* + iQ_i^*$ (today)



3. **coupling** via Kirchhoff & Ohm — injection $= \sum$ power flows

4. purely inductive lines $G/B \approx 0$ (can be relaxed to $G/B = const.$)

5. decoupling: $P_i \approx P_i(\theta)$ & $Q_i \approx Q_i(E)$ (near operating point)

- active power: $P_i \;=\; \sum_j B_{ij} E_i E_j \sin(\theta_i - \theta_j) + G_{ij} E_i E_j \cos(\theta_i - \theta_j)$
- reactive power: $Q_i \;=\; -\sum_j B_{ij} E_i E_j \cos(\theta_i - \theta_j) + G_{ij} E_i E_j \sin(\theta_i - \theta_j)$

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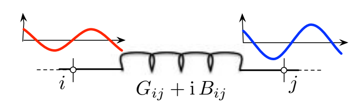

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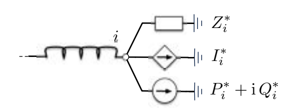

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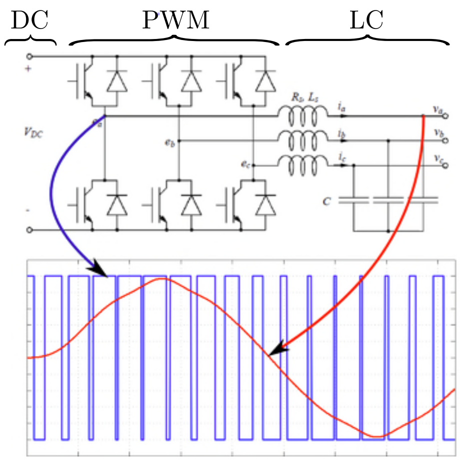
- trigonometric active power flow: $P_i(\theta) \;=\; \sum_j B_{ij} \sin(\theta_i - \theta_j)$
- polynomial reactive power flow: $Q_i(E) \;=\; -\sum_j B_{ij} E_i E_j$ (not today)

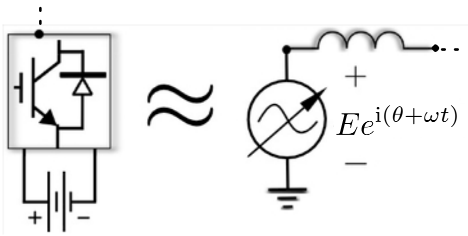
Modeling: sources interfaced with inverters

(all results also apply to synchronous machines & frequency-dependent loads)

Power **inverters** are ...

- **interfaces** between
  - ◊ the AC microgrid and
  - ◊ DC & variable AC sources
- **controllable** (voltage) sources
  [Zhong & Hornik, '12]

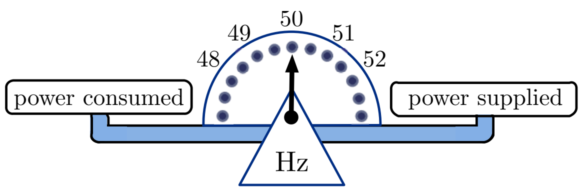




primary control

Decentralized primary control of active power

Inverters are controlled to emulate the physics of synchronous generators.
[Chandorkar et. al. '93]



Intuition: Recall...

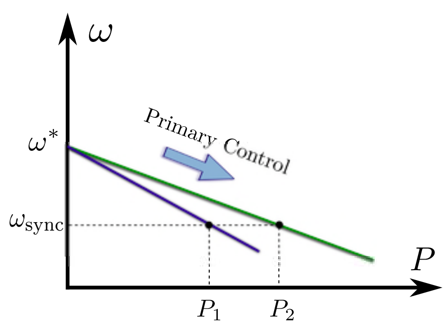
$P_i(\theta) = \sum_{j=1}^{n} B_{ij} \sin(\theta_i - \theta_j)$

$P/\dot\theta$ droop control:

$(\omega_i - \omega^*) \propto (P_i^* - P_i(\theta))$

⇕

$D_i\dot\theta_i = P_i^* - P_i(\theta)$



Putting the pieces together...

differential-algebraic closed loop

network physics

load power balance: $P_i^* = \sum_j B_{ij} \sin(\theta_i - \theta_j)$

source injections: $P_i(\theta) = \sum_j B_{ij} \sin(\theta_i - \theta_j)$

$\dot\theta_i$ $P_i(\theta)$

droop control

$\dot\theta_i = \frac{1}{D_i}(P_i^* - P_i(\theta))$

loads: $0 = P_i^* - \sum_j B_{ij} \sin(\theta_i - \theta_j)$

sources: $D_i\dot\theta_i = P_i^* - \sum_j B_{ij} \sin(\theta_i - \theta_j)$

Closed-loop stability under droop control

Theorem: stability of droop control [J. Simpson-Porco, FD, & F. Bullo, '12]

$\exists$ unique & exp. stable frequency sync $\iff$ active power flow is feasible

Main **proof ideas** and some **further results**:

- synchronization frequency: $\omega_{\text{sync}} = \omega^* + \dfrac{\sum_{\text{inverters}} P_i^* + \sum_{\text{loads}} P_i^*}{\sum_{\text{inverters}} D_i}$
  ($\propto$ power balance)

- steady-state power injections: $\mathcal{P}_i = \begin{cases} P_i^* & \text{for loads} \\ P_i^* - D_i(\omega_{\text{sync}} - \omega^*) & \text{for inverters} \end{cases}$
  (depend on D_i & P_i^*)

- unique steady-state branch flows: $\xi_{ij} = B_{ij} \sin(\theta_i^* - \theta_j^*) \Rightarrow \boxed{B_{ij} \geq \xi_{ij}}$
  ($\mathcal{P}_i \mapsto \xi_{ij}$)

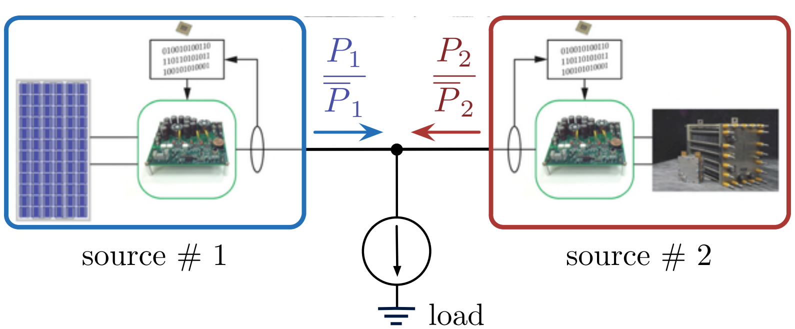
tertiary control

Objective I: decentralized proportional load sharing

1) Inverters have **injection constraints**: $P_i(\theta) \in [0, \overline{P}_i]$

2) Load must be **serviceable**: $0 \leq \left|\sum_{\text{loads}} P_j^*\right| \leq \sum_{\text{inverters}} \overline{P}_j$

3) **Fairness:** load should be shared proportionally: $P_i(\theta)/\overline{P}_i = P_j(\theta)/\overline{P}_j$



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Theorem: fair proportional load sharing [J. Simpson-Porco, FD, & F. Bullo, '12]

Let the droop coefficients be selected **proportionally**:

$$\boxed{D_i/\overline{P}_i = D_j/\overline{P}_j \;\; \& \;\; P_i^*/\overline{P}_i = P_j^*/\overline{P}_j}$$

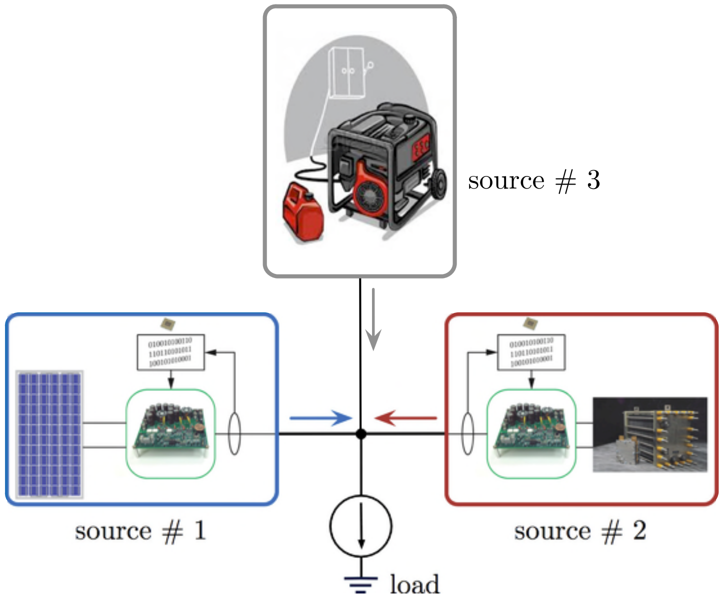
The the following statements hold:

(i) Proportional load sharing: $P_i(\theta)/\overline{P}_i = P_j(\theta)/\overline{P}_j$

(ii) Constraints met: $0 \leq \left|\sum_{\text{loads}} P_j^*\right| \leq \sum_{\text{inverters}} \overline{P}_j \;\Leftrightarrow\; P_i(\theta) \in [0, \overline{P}_i]$

Objective I: fair proportional load sharing

proportional load sharing is not always the right objective



Objective II: optimal economic dispatch

minimize the total accumulated generation

$$\text{minimize}_{\theta \in \mathbb{T}^n,\, u \in \mathbb{R}^{n_I}} \quad f(u) = \sum\nolimits_{\text{inverters}} \alpha_i u_i^2$$

subject to

inverter power balance: $P_i^* + u_i = P_i(\theta)$

load power balance: $P_i^* = P_i(\theta)$

branch flow constraints: $|\theta_i - \theta_j| \leq \gamma_{ij} < \pi/2$

inverter injection constraints: $P_i(\theta) \in [0, \overline{P}_i]$

Problem is generally non-convex and feasible only if the load is serviceable

In conventional power system operation, the economic dispatch is

- solved **offline**, in a **centralized** way, & with a **model & load forecast**

In an autonomously managed microgrid, the economic dispatch should be

- solved **online**, in a **decentralized** way, & **without knowing a model**

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Objective II: decentralized dispatch optimization

Insight: droop-controlled microgrid = decentralized primal algorithm

Theorem: optimal droop [FD, J. Simpson-Porco, & F. Bullo, '14]

The following statements are equivalent:

(i) the economic dispatch with cost coefficients α_i is **strictly** feasible with global minimizer (θ^*, u^*).

(ii) $\exists$ droop coefficients D_i such that the microgrid possesses a unique & locally exp. stable sync'd solution θ satisfying $P_i(\theta) \in [0, \overline{P}_i)$.

If (i) & (ii) are true, then $\theta_i \sim \theta_i^*$, $u_i^* = -D_i(\omega_{\text{sync}} - \omega^*)$, & $\boxed{D_i \alpha_i = D_j \alpha_j}$.

- similar results hold for the general **constrained** case
- similar results in transmission ntwks with DC flow [E. Mallada & S. Low, '13] & [N. Li, L. Chen, C. Zhao, & S. Low '13] & [X. Zhang & A. Papachristodoulou, '13] & [M. Andreasson, D. V. Dimarogonas, K. H. Johansson, & H. Sandberg, '13] & ...

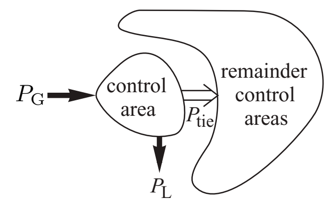
secondary control

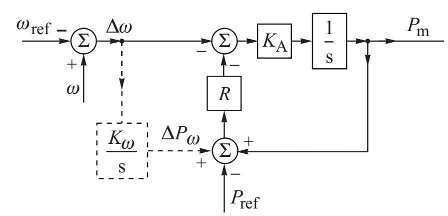
Secondary frequency control in power networks

Problem: steady-state frequency deviation ($\omega_{\rm sync} \neq \omega^*$)

Solution: integral control [Chandorkar et. al. '93, Lopes et al. '05, Bevrani '09, ...]

Interconnected Systems	Isolated Systems
• **Centralized** automatic generation control (AGC)	• **Decentralized** PI control





compatible with econ. dispatch [N. Li, L. Chen, C. Zhao, & S. Low '13]

is *globally* stabilizing [C. Zhao, E. Mallada, & FD, '14]

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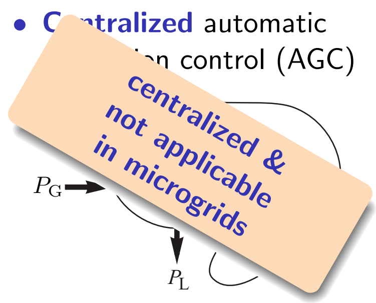

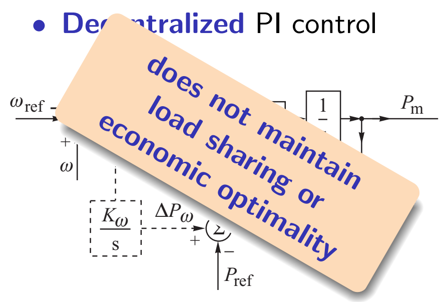

compatible with econ. dispatch

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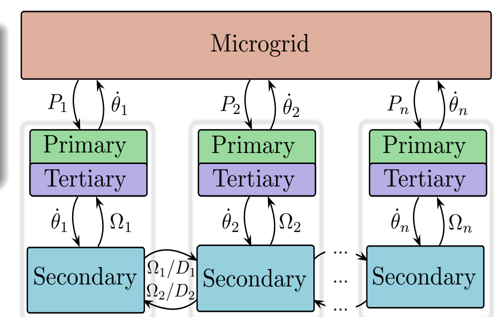
Microgrids require **distributed** (**!**) secondary control strategies.

Distributed Averaging PI (DAPI) control

$$D_i\dot{\theta}_i = P_i^* - P_i(\theta) - \Omega_i$$

$$k_i\dot{\Omega}_i = D_i\dot{\theta}_i - \sum_{j \subseteq \text{inverters}} a_{ij} \cdot \left(\frac{\Omega_i}{D_i} - \frac{\Omega_j}{D_j}\right)$$



- no tuning & no time-scale separation: $k_i, D_i > 0$
- distributed & modular: connected comm. $\subseteq$ inverters
- recovers primary op. cond. (load sharing & opt. dispatch)

$\Rightarrow$ plug'n'play implementation

Theorem: stability of DAPI
[J. Simpson-Porco, FD, & F. Bullo, '12]

primary droop controller works

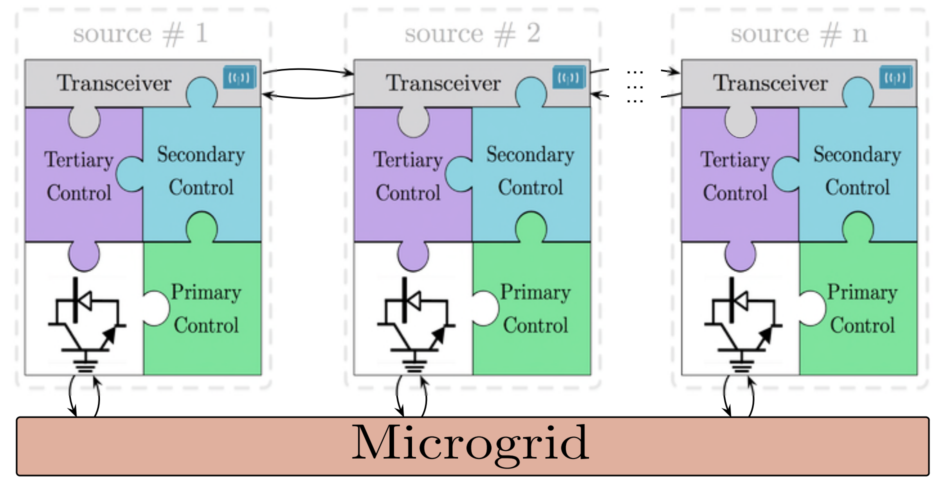
$\Longleftrightarrow$

secondary DAPI controller works

plug-and-play experiments

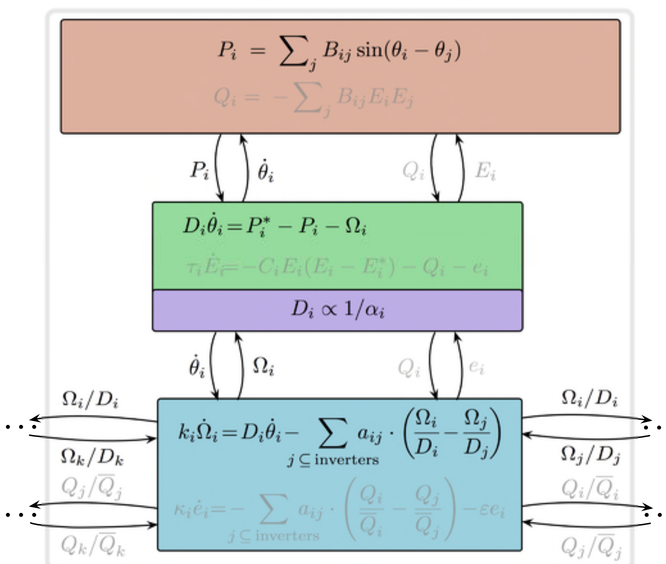
Plug'n'play architecture

flat hierarchy, distributed, no time-scale separations, & model-free



Plug'n'play architecture

recap of detailed signal flow (active power only)



Microgrid: physics & loadflow

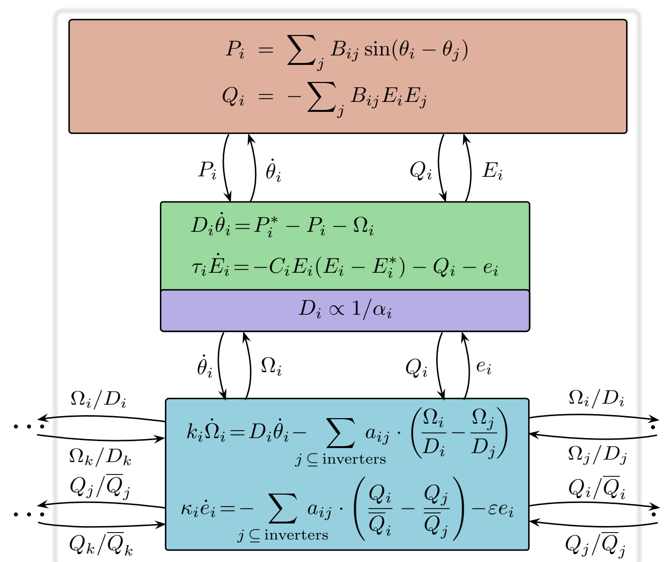
Primary control: mimic oscillators & polyn. symmetry

Tertiary control: marginal costs $\propto 1$ /control gains

Secondary control: diffusive averaging of injection ratios

Plug'n'play architecture

similar results in the reactive case



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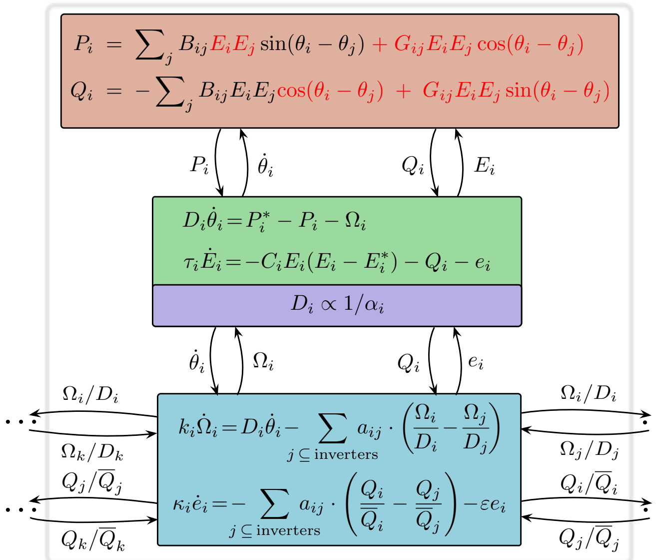
Plug'n'play architecture

experiments also work well in the coupled & lossy case



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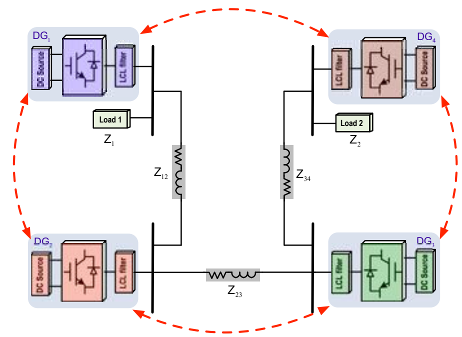
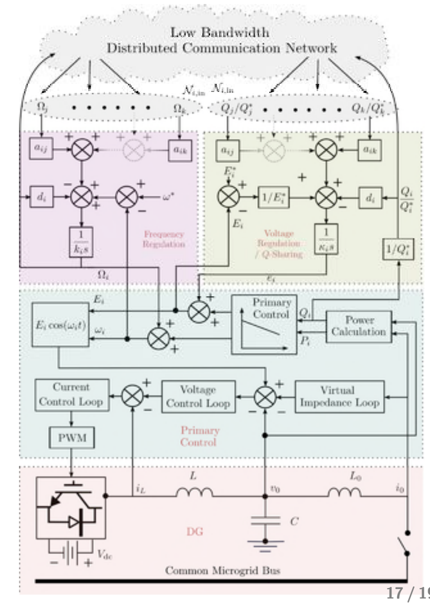
Experimental validation of control & opt. algorithms

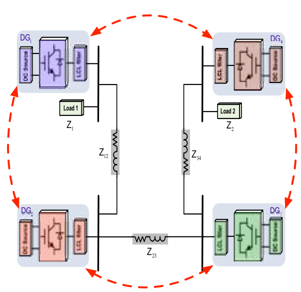
in collaboration with Q. Shafiee & J.M. Guerrero @ Aalborg University





Experimental validation of control & opt. algorithms

frequency/voltage regulation & active/reactive load sharing

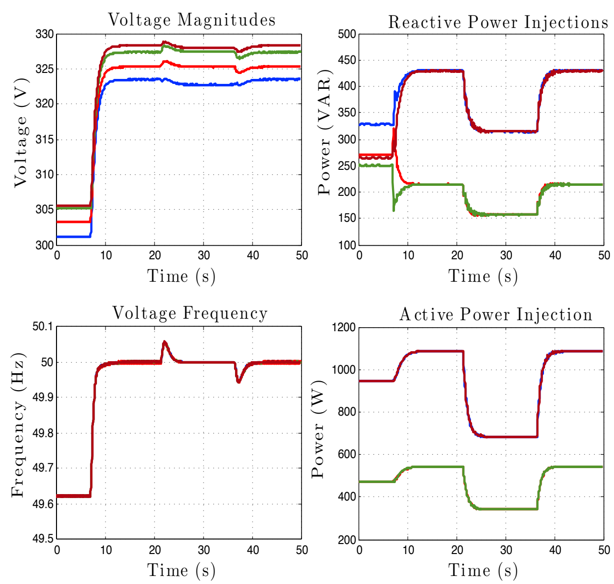


$t \in [0s, 7s]$: primary & tertiary control

$t = 7s$: secondary control activated

$t = 22s$: load # 2 unplugged

$t = 36s$: load # 2 plugged back



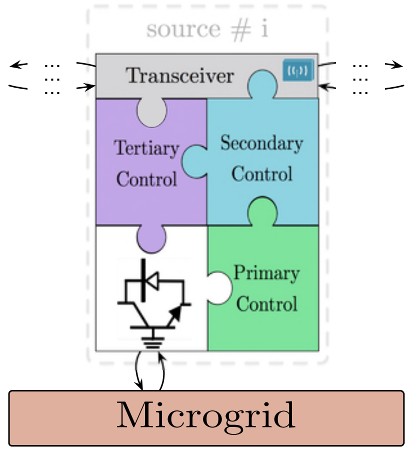
conclusions

Conclusions

Summary
- primary $P/\dot{\theta}$ droop control
- fair proportional load sharing & economic dispatch optimization
- distributed secondary control strategies based on averaging
- experimental validation

Further results
- reactive power control
- virtual oscillator control

Open conjecture
- solve these problems without comm



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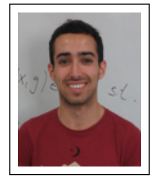
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